DOCUMENT RESUME

ED 121 590 SE 020 460

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TITLE Study Modules for Calculus-Based General Physics. [Includes Modules 38-40: Optical Instruments; Diffraction; and Alternating Current Circuits].
INSTITUTION Nebraska Univ., Lincoln.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 75
NOTE 75p.; Prepared at a College Faculty Workshop (University of Colorado, Boulder, June 23-July 11, 1975); Directions for the CBP modules are found in SE 020 448; Related documents are SE 020 449-461
AVAILABLE FROM University of Nebraska, CBP Workshop, Behlen Laboratory of Physics, Lincoln, Nebraska 68588 (entire set of 42 modules, $15.00)

EDRS PRICE MF-$0.83 HC-$3.50 Plus Postage
DESCRIPTORS *Calculus; College Science; *Course Descriptions; Electricity; Higher Education; Individualized Curriculum; Individualized Instruction; Instructional Materials; Optics; *Physics; *Programed Instruction; Science Education; Undergraduate Study
IDENTIFIERS Keller Plan; *Personalized System of Instruction; PSI

ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 38--Optical Instruments, Module 39--Diffraction, and Module 40--Alternating Current Circuits. (CP)

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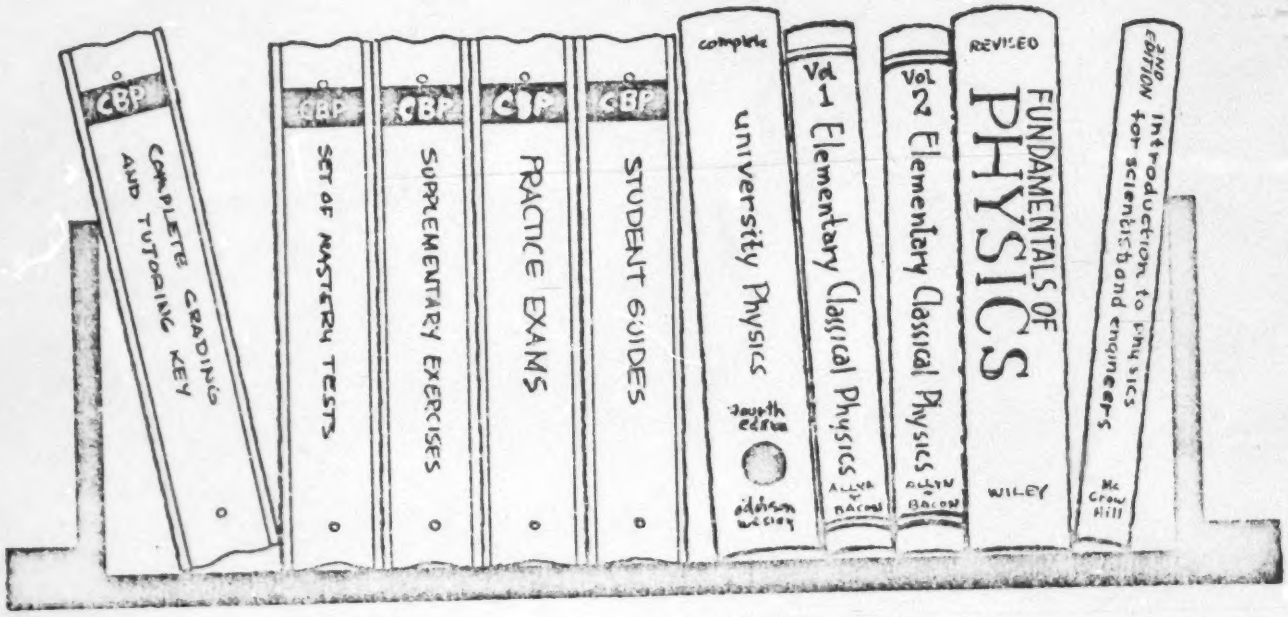
STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



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SE 020 460

Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

Robert G. Fuller
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These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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STUDY GUIDE

OPTICAL INSTRUMENTS

INTRODUCTION

You are now familiar with some of the properties of idealized single lenses and simple spherical and plane mirrors. Almost all optical instruments are made up of a combination of lenses, some close together and others far apart. Real lenses and mirrors have many undesirable properties intimately interconnected with their desirable properties. By making careful and clever combinations of lenses one can enhance the desirable and minimize the undesirable characteristics. In this module you will begin the study of some simple combinations of mirrors and simple lenses; it will give you some insight into the complications and fascinating possibilities of complex optical systems.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Solve problems to find image or object location and lateral magnification by calculation and by ray diagram for spherical mirrors and thin lenses (needed for Objectives 1, 2, and 4 of this module)	Lenses and Mirrors Module

LEARNING OBJECTIVES

When you have mastered the content of this module, you will be able to:

1. Ray diagrams - Use ray diagrams to find magnification and locate objects, images, or lenses when appropriate other information is given for two-lens systems.

2. Lens equation - Use the thin-lens equation to find magnification and locate objects, images, or lenses when appropriate other information is given for two-lens systems.

3. Angular magnification - (a) Define angular magnification for various optical instruments. (b) Apply the definition of angular magnification to systems of two lenses, or one lens and one mirror, such as telescopes and compound microscopes.

STUDY GUIDE: Optical Instruments 2(B 1)

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Study the text Chapter 33, Sections 33.1, 33.3, and 33.4, and read the General Comments. Then study Problems A through F and work the Assigned Problems. To satisfy Objective 1 you should draw ray diagrams for the assigned problems even though they are not specifically requested. Study the ray diagrams in the study guide and Figure 33.5(b) in the text, but note that Figures 33.1, 33.4, and 33.5(a) are not ray diagrams. There is a minor notation problem that the following table should help.

	Text	Study Guide
Index of refraction	μ	n
Object distance	p	s
Image distance	p′	s′

BUECHE

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text (Chap. 33)	(Chap. 33)
1	Sec. 33.4	A, B	G		
2		C, D	G	13, 15, 17	14, 18
3	Secs. 33.1, 33.3, 33.4	E, F	H	8, 9, 11, 12	

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Since there is no discussion in your text specifically covering the objectives of this module, you can follow the study guide supplied by your tutor if you have one of the following texts available.

Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition.

Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition. (Note that Sears and Zemansky use s and s′, instead of o and i for object and image distance, respectively.) Note also that most of the diagrams are not ray diagrams that follow the ray-construction rules.

STUDY GUIDE: Optical Instruments 2(SZ 1)

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Study the text Chapter 40, Sections 40-5, 40-10, 40-13, and 40-14, and read the General Comments of the study guide. Study Problems A through F and ray diagrams, noting that most of the diagrams in the text are not strictly ray diagrams that follow the ray-construction rules. Work the Assigned Problems. To satisfy Objective 1, you should draw ray diagrams for all Assigned Problems, even though they are not specifically requested.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1		A, B	G		
2	Sec. 40-5	C, D	G	40-20, 40-21(a), 40-22	
3	Secs. 40-10, 40-13, 40-14	E, F	H	40-32, 40-33, 40-37, 40-38, 40-40	40-35, 40-39

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Study the text Chapter 37, Section 37-3 on lens combinations, and be sure to study Figures 37-16 through 37-20, as they are carefully drawn and merit careful study although little is said about them in the text. Read the General Comments. Study Problems A through F and work the Assigned Problems. You should draw ray diagrams for the assigned text problems even though it is not specifically suggested in the problem.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Sec. 37-3	A, B	G	
2	Sec. 37-3	C, D	G	37-11, 37-12, 37-14
3	Sec. 37-3	E, F	H	37-17

GENERAL COMMENTS

Anyone who has a modest competance in geometrical optics has found ray diagrams essential for the understanding of optical systems. You have probably already discovered that remembering and correctly applying the sign conventions for the thin-lens equation can at times be difficult, and it gets worse for multiple-component systems. Drawing a ray diagram has fewer and less confusing rules. Ray diagrams provide a relatively independent check on the results of a calculation using the lens equations. Successful use of a ray diagram does require some practice, and you must draw several before trying a Mastery Test.

There is one case of two thin lenses that is particularly simple - when they are close enough together so that you can neglect the distance between them and when the object distance for the second lens is the negative of the image distance for the first lens. One of the Assigned Problems asks you to work out the details.

One experimental way to find the focal length of a converging lens is to set up an object and the lens and then find the location of the real image on a screen. A simple calculation using the object distance, image distance, and the thin-lens equation will then give you the focal length. This method cannot be used directly with a diverging lens because with normal diverging wavefronts a diverging lens cannot form a real image. Problems B and D illustrate a method of getting a real image with a diverging lens and thus a way to find its focal length.

Our ability to distinguish details depends in part upon the size of the image on our retina, which in turn depends on the angle subtended by the object at our eye. Thus to see an object more clearly we bring it closer to our eyes. There is a limit, however, because when it gets too close we can no longer focus on the object. We somewhat arbitrarily take 25 cm as the nearest distance of distinct vision, and characterize many optical instruments by their angular magnification. For a microscope, angular magnification (M) is defined as the ratio of the angle subtended by the image viewed by the eye to the angle subtended by the object at 25 cm. For a telescope, however, the angular magnification is defined as the ratio of the angle subtended by the image viewed by the eye to the angle subtended by the object at its actual location. These definitions are slightly different but reflect the actual use of the instrument. Note that the actual magnification will depend on the location of the final image, which in most instruments is adjustable.

PROBLEM SET WITH SOLUTIONS

A(1). Two thin converging lenses are on a common axis 1.00 m apart. The first lens (L_1) has a focal length $f_1 = +0.40$ m; the second lens (L_2) has a focal length $f_2 = +0.200$ m. An object 0.0200 m high is 0.90 m to the left of the first lens. Find the location and size of the final image by making a ray diagram.

Solution

To solve this problem draw a ray diagram, as you learned in the module Lenses and Mirrors, to find the image of the object as formed by the first lens. This is Figure 1. Now using the image formed by the first lens L_1 as the object for the

second lens, draw a ray diagram for the second lens, as in Figure 2. There is no need to use extensions of any of the rays from the ray diagram of L_1. Ray diagrams in textbooks usually show particular rays continuing through both lenses, but that is not necessary.

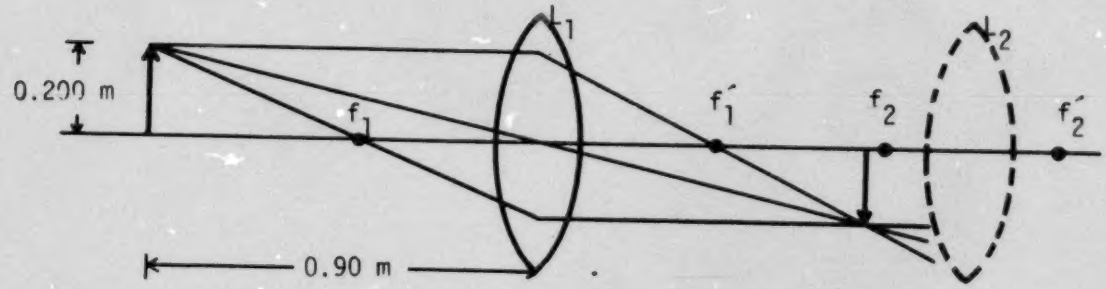


Figure 1

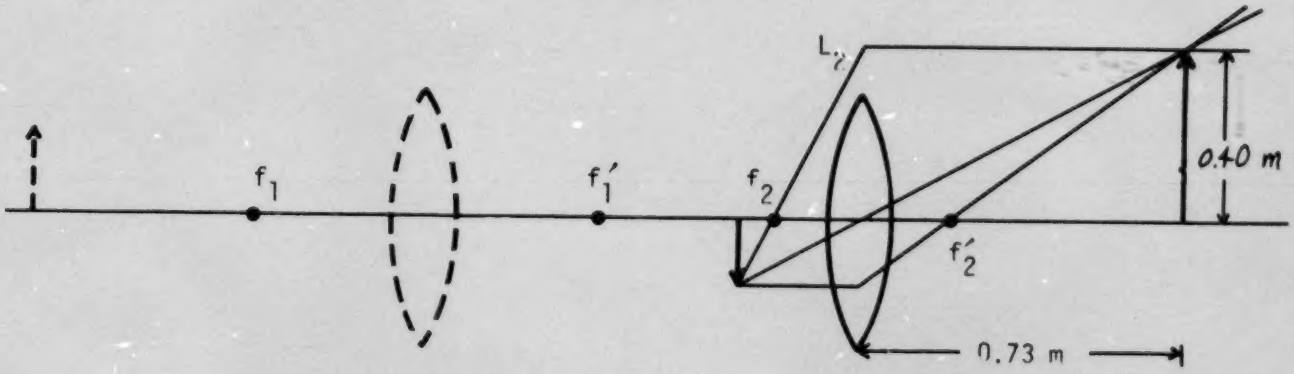


Figure 2

B(1). A thin converging lens (L_1) of focal length f_1 = 0.40 m and a thin diverging lens (L_2) of focal length f_2 = -0.300 m are on a common axis 1.00 m apart. An object 0.0100 m high is 0.60 m to the left of L_1. Find the location and size of the final image by making a ray diagram.

Solution

As in Problem A find the image formed by the first lens: See Figure 3. The next step is a bit tougher, however: we cannot treat the image from L_1 as a simple

object for L_2 because its rays are intercepted by L_2 before they can form an image (virtual object). We shall proceed by choosing very carefully the rays that we will draw.

(1) A parallel ray headed toward a virtual object point will diverge from f_2.

(2) A ray headed for f_2' and a virtual object point will be refracted parallel to the axis.

(3) A ray passing through the center of the lens and heading for a virtual object point will be undeflected.

Note that on a ray diagram, you may use rays refracted by an extension of the lens. See Figure 4.

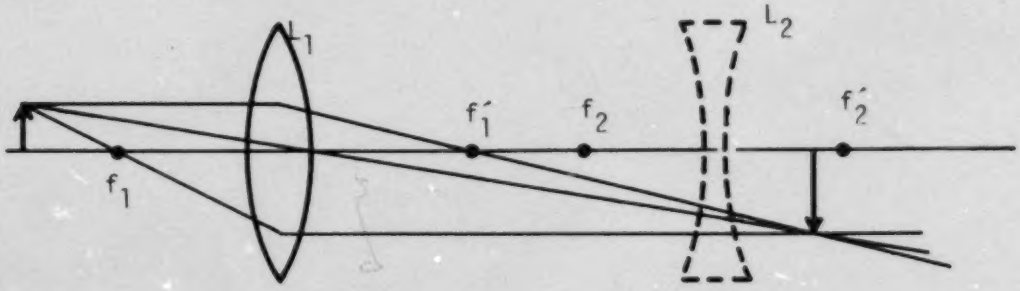


Figure 3

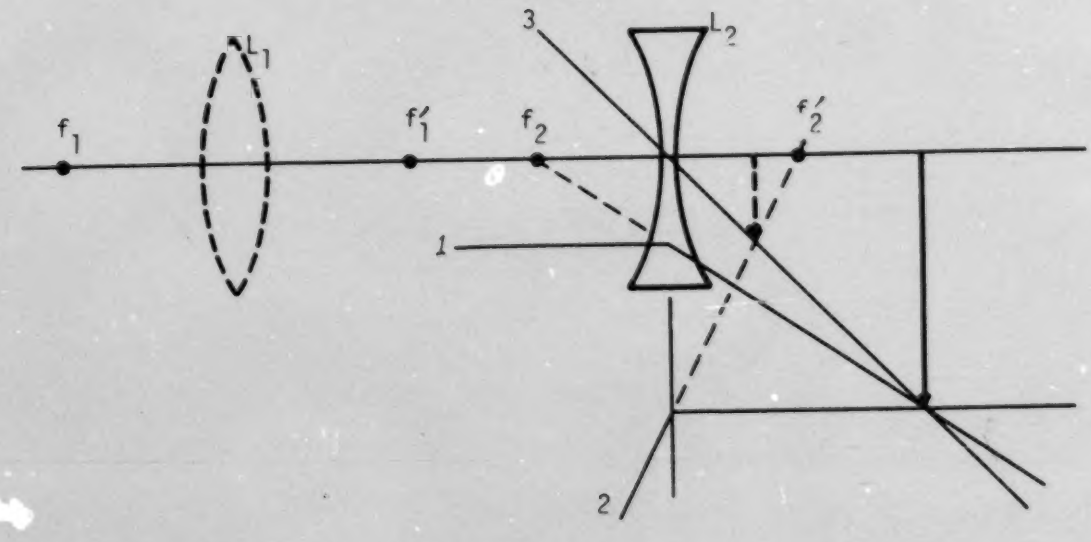


Figure 4

C(2). For the same conditions as in Problem A, find the location and size of the final image by calculation.

Solution

Apply the approximate thin-lens equation to find the image location of the original object as formed by the first lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_1}, \qquad \frac{1}{0.90} + \frac{1}{s'} = \frac{1}{0.40}, \qquad s' = 0.72 \text{ m}.$$

From similar triangles on the ray diagram or the formula for lateral magnification,

$$\text{image size} = -(s'/s)(\text{object size}) = (0.72/0.90)(0.0200) = -0.0160 \text{ m}.$$

The negative simply implies an inverted image. Since the lenses are 1.00 m apart, the image from L_1 is now the object for L_2 and has an object distance

$$s_2 = 1.00 - 0.72 = 0.280 \text{ m}.$$

We now apply the thin-lens equation again:

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}, \qquad \frac{1}{0.280} + \frac{1}{s_2'} = \frac{1}{0.200}, \qquad s_2' = 0.70 \text{ m},$$

$$\text{final image size} = -(-0.0160)(0.70/0.280) = 0.040 \text{ m}.$$

These results agree quite closely with the graphical solution to Problem A.

D(2). Work Problem B using the thin-lens equation.

Solution

The location and size of the image formed by the first lens should be fairly routine by now:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_1}, \qquad \frac{1}{0.60} + \frac{1}{s'} = \frac{1}{0.40}, \qquad s' = 1.20 \text{ m};$$

$$\text{image height} = -(s'/s)(\text{object height}) = -\frac{1.20}{0.60}(0.0100) = -0.0200 \text{ m}.$$

This image now becomes the object for L_2, its object distance

$$s_2 = 1.00 - 1.20 = -0.200 \text{ m}.$$

The negative sign indicates a virtual object.

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}, \qquad \frac{1}{0.200} + \frac{1}{s_2'} = -\frac{1}{0.300}, \qquad s_2' = 0.60 \text{ m};$$

$$\text{image height} = -(s'/s)(\text{object height}) = -[0.60/(-0.200)](-0.0200) = -0.060 \text{ m}.$$

This result agrees with the result from the lens diagram in Problem B.

E(3). The Moon at a distance of 3.8×10^8 m subtends an angle of $\sim 0.5°$ as seen from the Earth. With a telescope objective lens L_1 of focal length 2.00 m:
(a) Find the image I_1, location, and size.
(b) A converging lens L_2 (f = 5.0 cm) is used to view the image I_1 created by the objective lens. The final image I_2 is virtual and 0.250 m from L_2. Find the distance from image I_1 to L_2.
(c) What angle does the image I_1 subtend at the lens L_2?
(d) What angle does the image I_2 subtend at the lens L_2?
(e) What is the angular magnification of this simple telescope?

Solution

(a) From the thin-lens equation, since the object distance is very large the image will be at f = 2.00 m. The image and object of a thin lens subtend equal angles; thus, $\theta = y/f$, where y is the image size:

$$y = f\theta = (2.00 \text{ m})(0.5°) = (2.00 \text{ m})(0.00873 \text{ rad}) = 0.0175 \text{ m}.$$

(b) For L_2, s_2 is unknown. $s' = -0.250$ m, f = 0.050 m,

$$\frac{1}{s_2} - \frac{1}{0.250} = \frac{1}{0.050}, \qquad s_2 = 0.042 \text{ m}.$$

(c) The angle subtended,

$$\tan \theta_2 = y/s_2 = 0.0174/0.042 = 0.42, \qquad \theta_2 = 0.40 \text{ rad} = 22.7°.$$

Since we have frequently used the small-angle approximation, we could equally as well have said $\theta_2 = y/s_2 = 0.42$.

(d) The image subtends the same angle as the object:

$$\theta_2' = \theta_2 = 0.40 \text{ rad} = 22.7°.$$

(e) $M = \theta_2'/\theta = 22.7/0.50 = 45$. Notice that this corresponds roughly to the approximate formula

$$M = f_o/f_e = 2.00/0.050 = 40$$

(derived for the final image at infinity).

F(3). A compound microscope has an objective lens L_1, of focal length 4.0 mm. It forms an image I_1 at a distance of 16.0 cm of an object of 10^{-5} m diameter. It is viewed by a simple eyepiece (L_2, f_2 = 2.50 cm) adjusted to form a virtual image I_2 at infinity.

(a) Find the angle subtended by the object at the closest distance of distinct vision.

(b) How large is the image I_1?

(c) What angle does the image I_1 subtend at L_2 when the final image I_2 is at infinity?

(d) What angle does I_2 subtend at L_2?

(e) Find the angular magnification of the microscope.

Solution

(a) $\tan\theta \approx \theta = 10^{-5}/0.250 = 4.0 \times 10^{-5}$ rad.

(b) The image size = (object size)$(s_1'/s_1) = 10^{-5}(0.160/0.0041) = 3.9 \times 10^{-4}$ m. The object distance will be slightly larger than f_1:

$$\frac{1}{s_1} + \frac{1}{0.160} = \frac{1}{0.0040}, \qquad s_1 = 0.0041.$$

(c) For I_2 at infinity s_2, the object distance, is f_2 = 0.025 m. Thus,

$$\theta_2 = (3.9 \times 10^{-4})/0.0250 = 0.0156 \text{ rad.}$$

(d) Same as part (c), 0.0156 rad.

(e) $M = \theta_2/\theta = 0.0156/(4.0 \times 10^{-5}) = 390.$

Problems

G(1, 2). A thin diverging lens, f_1 = -0.40 m, and a thin converging lens, f_2 = 0.200 m, are on a common axis 0.100 m apart. An object 0.040 m high is located 0.40 m to the left of the diverging lens. Find the location and size of the final image by a ray diagram and by calculation from the thin-lens equation.

H(3). A Galilean telescope is to be constructed using an objective lens L_1 of f_1 = +0.40 m and a diverging lens L_2 of focal length f_2 = -0.050 m.

(a) What angle is subtended by a 2.00-m object at 100 m distance?

(b) How large an image will be formed by lens L_1?

(c) The diverging lens L_2 is placed so that the final image I_2 is a virtual image at infinity. Find the angle subtended by the virtual object I_1 at L_2.

(d) Find the angle subtended by I_2 at L_2.

(e) Find the angular magnification M for the Galilean telescope adjusted as above.

Solutions

G(1, 2). Location: 0.60 m to the right of the converging lens. Size: 0.040 m high.

H(3). 0.0200 rad. (b) 0.0080 m. (c) 0.160 rad. (d) 0.160 rad. (e) 7.9.

PRACTICE TEST

1. Two thin lenses each of focal length +20.0 cm are 20.0 cm apart. An object is 10.0 cm to the left of the left-hand lens. Find the location of the final image by both a ray diagram and calculation using the thin-lens equation.

2. Define angular magnification for a microscope.

3. An opera glass consists of a converging lens of focal length 0.60 m and a diverging lens of focal length -0.100 m. Find the angular magnification for a 2.00-m tall opera singer at a distance of 30.0 m. (If you use any simple formulas for angular magnification be sure that you derive them.) Assume the final image to be at infinity.

Practice Test Answers

1. 40 cm right of right-hand lens.

2. The ratio of the angle subtended by the final image at the ocular, to the angle subtended by the object at 25 cm.

3. 5.8.

OPTICAL INSTRUMENTS Date _____

Mastery Test Form A

pass recycle

1 2 3

Name _____ Tutor _____

1. Two thin lenses each of focal length +0.200 m are 0.200 m apart. An object is 0.200 m to the left of the left lens. Find the location of the final image both by a ray diagram and by a calculation using the thin-lens equation.

2. Define angular magnification for a microscope.

3. The telescope in Figure 1 is constructed from an objective that is a concave spherical mirror of diameter 0.200 m and radius of curvature 3.00 m, and an eyepiece equivalent to a simple lens of focal length 0.0250 m. The telescope is used to view a pair of stars whose separation subtends an angle of 0.00100 rad. The final image of the star pair is formed by the telescope at 1.00 m. Find the angular magnification of the telescope when so adjusted. (Do not use any simple formulas you may have remembered, but do the calculation step by step from the definition.) The plane mirror simply reflects the image out of the incident beam and need not be considered in this problem.

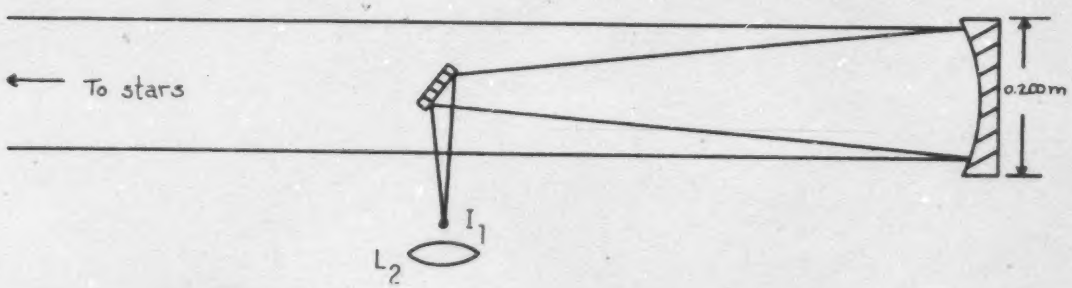


Figure 5

OPTICAL INSTRUMENTS Date _____

Mastery Test Form B

pass recycle

1 2 3

Name _____ Tutor _____

1. Two thin lenses each of focal length +0.200 m are 0.200 m apart. An object is 0.40 m left of the left lens. Find the location of the final image both by a ray diagram and by a calculation using the thin-lens equation.

2. Define angular magnification for a Galilean telescope.

3. A compound microscope is constructed from two simple converging lenses of focal lengths 5.0 mm (objective) and 5.0 cm (eyepiece) and a tube, separating the lenses, 0.200 m long. The object is located so that the final image is at infinity. Find the angular magnification. (Do not use any simple formulas you may have remembered, but do the calculation step by step from the definition.)

OPTICAL INSTRUMENTS Date _____

Mastery Test Form C

pass recycle

1 2 3

Name _____ Tutor _____

1. Two thin lenses, the one on the left of focal length -0.200 m and the second of focal length +0.200 m, are separated by 0.200 m. An object is 0.200 m to the left of the left lens. Find the location of the final image both by a ray diagram and by a calculation using the thin-lens equation.

2. Define angular magnification for a telescope.

3. A pair of binoculars has an objective lens of focal length of 0.280 m and an eyepiece of effective focal length 0.040 m. They are focused on an object 0.050 m high at a distance of 6.0 m with the final image at 0.250 m. Find the angular magnification. (Do not use any simple formulas that you may have remembered, but do the calculations step by step from the definition.)

MASTERY TEST GRADING KEY - Form A

1. What To Look For: The ray diagram is a bit tricky, but the student should indicate parallel rays between the lenses (parallel to each other, not the axis). Check sign, magnitude, and units of the numerical answer.

   Solution: See Figure 6. For the first lens:

   First lens: $1/0.200 + 1/s_1' = 1/0.200, \quad s_1' = \infty.$

   Second lens: $1/\infty + 1/s_2' = 1/0.200, \quad s_2' = 0.200$ m to right of right lens.

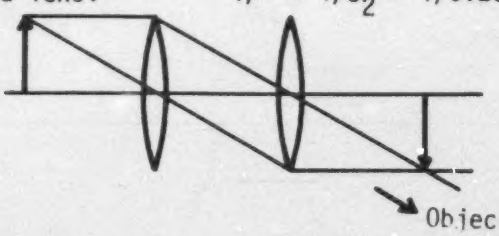


Figure 6

2. Solution: The angular magnification of a microscope is the ratio of the angle subtended by the final image at the ocular to the angle subtended by the object at the nearest distance of distinct vision, usually 25.0 cm.

3. What To Look For: Be sure that the problem is solved in a stepwise fashion similar to that below, and not by $M = f_o/f_e$.

   Solution: The focal length of a spherical mirror is $R/2 = f = 1.50$ m. For an object as far away as a star the image will be at the focal plane. The separation of star images will be

   $d = f\theta_1 = (1.50)(0.00100) = 1.50 \times 10^{-3}$ m.

   To find the distance to the ocular we use the thin-lens equation:

   $1/s_2 - 1/1 = 1/0.0250, \quad s_2 = 0.0246$ m.

   $\theta_2 = d/s_2 = (1.50 \times 10^{-3})/0.0246 = 0.061$ rad.

   $M = 2/0.00100 = 0.061/0.00100 = 61.$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: Be sure that the ray diagram has rays that can be constructed and is not "faked" from the numerical answer. Check magnitude, units, and reference point of final answer.

   Solution: See Figure 7. $1/0.40 + 1/s_1' = 1/0.200$, $s_1' = 0.40$ m;

   $s_2 = 0.200 - s_1' = 0.200 - 0.40 = -0.200$;

   $-1/0.200 + 1/s_2' = 1/0.200$, $s_2' = 0.100$ m to right of right lens.

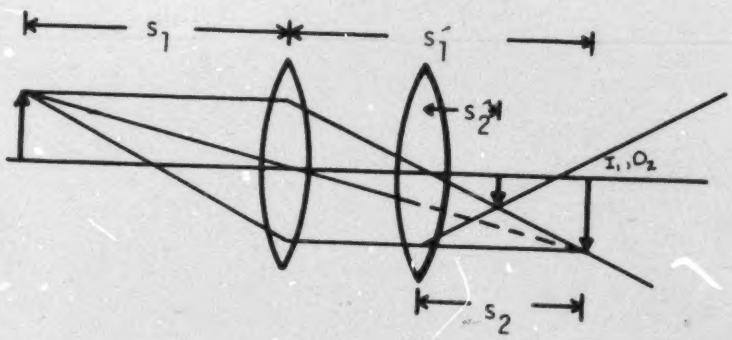


Figure 7

2. Solution: The angular magnification of a telescope is the ratio of the angle subtended by the final image at the ocular to the angle subtended by the object at the objective lens.

3. What To Look For: Be sure that the problem is done stepwise similar to the Solution below, and not by substitution into $M = L(25/f_o)f_e$ (except perhaps as a check).

   Solution: Assume an object y (small), $\theta_1 = y/0.250 = 4y$. Since the final image is at infinity its object is f = 0.050 m from the objective, 0.200 - 0.050 = 0.150, and the object distance can be calculated:

   $1/s_1 + 1/0.150 = 1/0.0050$, $s_1 = 0.0052$ m;

   image size $= y(0.150/0.0052) = 29y$.

   $\theta_2 = 29y/0.050 = 580y$; and, finally, $M = 580y/4y = 145$.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: Be sure ray diagram uses correct principal rays and is not "faked" from numerical answer. Check magnitude, units, and reference point of final answer.

   Solution: See Figure 8.

   $1/0.200 + 1/s_1' = -1/0.200$, $s_1' = -0.100$ m; $s_2 = 0.100 + 0.200 = 0.300$,

   $1/0.300 + 1/s_2' = 1/0.200$, $s_2' = 0.60$ m to right of right lens.

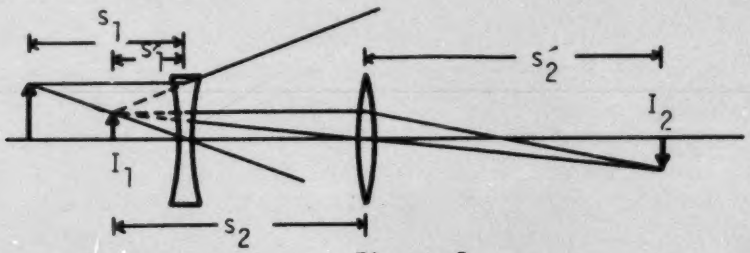


Figure 8

2. Solution: The angular magnification of a telescope is the ratio of the angle subtended at the ocular by the final image to the angle subtended by the object at the objective lens.

3. What To Look For: Be sure that the problem is done stepwise similar to the Solution below and not simply $M = f_o/f_e = 7$ (except maybe as a check on the result).

   Solution: The object subtends an angle $\theta_1 = 0.050/6 = 0.0083$ rad. The image is at s_1':

   $1/6 + 1/s_1' = 1/0.280$, $s_1' = 0.290$ m.

   Its size is $(0.050)(0.290/6) = 0.00240$ m. Its distance from the ocular is s_2:

   $1/s_2 - 1/0.250 = 1/0.040$, $s_2 = 0.0340$ m.

   The angle subtended at the ocular is

   $\theta_2 = 0.00240/0.034 = 0.072$. $M = \theta_2/\theta_1 = 8.5$.

DIFFRACTION

INTRODUCTION

Have you ever wondered why you can hear around corners, but cannot see around them? You know that light and sound are waves, and should therefore share the same basic properties. Why then do they seem so different in the property of their "shadows"?

In this module you will learn that light does exhibit all the bending properties of sound and water waves. The effect, however, depends on the size of the obstacle compared to the wavelength. It is only the largeness of everyday obstacles compared to the very small wavelength of light that deemphasizes the bending, or diffraction, of light. In this module we shall use very small obstacles and windows in order to make the diffraction effect most apparent to you.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Explain interference of light in terms of the properties of waves (needed for Objectives 1 through 4 of this module)	Interference Module
*Give the source and wavelength of some common electromagnetic waves (needed for Objectives 1 through 4 of this module)	Wave Properties of Light Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Huygens' principle - Use Huygens' principle to explain how light from a single slit can produce interference fringes.

2. Fraunhofer diffraction - (a) State the optical conditions necessary to produce Fraunhofer diffraction through a single slit. (b) Use the equation for the diffraction intensity pattern from single-slit Fraunhofer diffraction to solve for the intensity, the position of various intensities, the size of the single slit, or the wavelength of the wave.

3. Resolving power - (a) Describe the conditions for which two objects viewed through a slit or circular aperture are just resolved. (b) Solve for the

separation, distance, or the wavelength emitted by two objects that are just resolved; or solve for the smallest orifice through which they can be identified as two objects.

4. Diffraction grating - Solve diffraction-grating problems that ask for the position of the principal (or most intense) maxima, the order number, the wavelength of the light, or the optical construction of the diffraction grating.

STUDY GUIDE: Diffraction 3(B.1)

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read General Comments 1 and 2 in this study guide, and Section 32.1 of Chapter 32. Then study Problem A. Next read Sections 32.2 through 32.4 and study Illustration 32.1 and Problem B, before working Problems E and F. Read Section 32.5 and study Problem C; work Problem G. Then read Sections 32.6 and 32.7, study Problem D, and work Problems H and I.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

BUECHE

Objective	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 32)
1	General Comments 1, 2, Sec. 32.1	A			Quest.[a] 1
2	Secs. 32.1 to 32.4	B	Illus.[a] 32.1	E, F	Quest. 4, 7, Probs. 1 to 8, 10
3	Sec. 32.5	C		G	Quest. 9, Prob. 9
4	Sec. 32.7	D		H, I	Quest. 2, 3, Probs. 14 to 16, 19, 21

[a]Illus. = Illustration(s). Quest. = Question(s).

STUDY GUIDE: Diffraction 3(HR 1)

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read General Comments 1 and 2 in this study guide, and Section 38-1 in Chapter 38. Then study Problem A. Read Sections 38-2 through 38-4 and study Problem B and Examples 1 to 3, before working Problems E and F. Read Section 38-5, study Problem C and Example 4, and work Problem G. Then read Sections 38-6 through 38-9, study Problem D and Example 7, and work Problems H and I.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 38)
1	General Comments 1, 2, Sec. 38-1	A			
2	Secs. 38-1 to 38-4	B	Ex.[a] 1, 2, 3	E, F	Quest.[a] 1 to 6, Probs. 1 to 4, 6, 7
3	Sec. 38-5	C	Ex. 4	G	Probs. 10 to 15
4	Secs. 38-7 to 38-9	D	Ex. 7	H, I	Quest. 14, 15, 17, Probs. 30 to 36

[a]Ex. = Example(s). Quest. = Question(s).

STUDY GUIDE: Diffraction 3(SZ 1)

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read General Comments 1 and 2 in this study guide, and Section 41-10 in Chapter 41. Then study Problem A. Next read Section 41-11, study Problem B, and work Problems E and F. Read Sections 41-12 and 41-13, and study Problem D and Examples 1 and 2. Then read Section 41-14 and study Problem C. Work Problems G through I.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comments 1, 2, Sec. 41-10	A			
2	Secs. 41-10, 41-11	B			41-15, 41-16, 41-17
3	Sec. 41-14	C		G	41-22, 41-23
4	Secs. 41-12, 41-13	D	Ex.[a] 1, 2 (Sec. 41-12)	H, I	41-18, 41-20, 41-21

[a]Ex. = Example(s)

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Since your text does not follow the order of the Learning Objectives of this module, you might do well to read Chapter 39 through quickly at first, for an overview, then study the sections according to objective, as given in the Table. Read General Comments 1 and 2 in this study guide. Then read Sections 39-1 through 39-3, study Problems A and B and Example 39-1, before working Problems E and F. Next read Sections 39-4 through 39-7, study Problems C and D and Example 39-2, and work Problems G, H, and I.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

WEIDNER AND SELLS

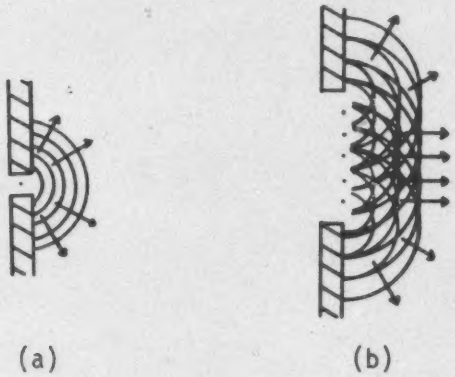
Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comments 1, 2, Secs. 39-3, 39-1	A			
2	Secs. 39-3, 39-1, 39-2	B	Ex.[a] 39-1	E, F	39-5, 39-7, 39-8, 39-9
3	Sec. 39-7	C		G	39-14, 39-15, 39-16, 39-17
4	Sec. 39-5	D	Ex. 39-2	H, I	39-19, 39-20

[a]Ex. = Example(s).

GENERAL COMMENTS

1. Huygens' Principle

Huygens' principle describes the motion of a wave moving away from its source by having you visualize that each point on the expanding wavefront is a source of spherical waves of the same wavelength. In Figure 1 are shown two examples of waves coming through different-size windows. The small left-hand window in Figure 1(a) can be thought of as containing only one point, with the spherical wave emanating from it and spreading into the region to the right of the window with equal intensity in all directions.



(a) (b)

Figure 1

Figure 2

As the window becomes larger, as in Figure 1(b), the transmitted wave becomes more planar and the bending around the edge becomes a less important effect. The many spherical waves produce interference in all directions except that normal to the plane of the windows. This is a qualitative reason why large windows (or obstacles) appear to cast sharp shadows.

2. Interference of Waves

All through the readings in this module you will encounter the light and dark fringes that you have learned to associate with interference of waves. This remains the correct interpretation of these fringes, but now the interfering waves are the spherical waves predicted by Huygens' principle. An example is shown in Figure 2. Two points, A and B, are shown on a wavefront moving to the right. The secondary waves from A and B are also shown. At point P the waves from A and B have different length paths and will interfere. Depending on what the pathlengths to P are for the other secondary waves from the wavefront, you might see a fringe at P.

It is not difficult to realize, though, that for a wavefront containing many points, the conditions for all the secondary waves to interfere in just the right way to produce fringes at P are rare. Your everyday experience agrees with this: except for cases such as when you are looking through an umbrella or at a far-away light, you do not see fringes.

PROBLEM SET WITH SOLUTIONS

A(1). A single slit is illuminated from the left with monochromatic waves, as in Figure 3. Use Huygens' principle to explain how fringes can be seen on the screen. Assume plane waves.

Solution

Pick points A and B on the edge of and midpoint of the slit. Draw some secondary Huygens' waves, as shown in Figure 4. Now that you have the spherical waves you can pick two parallel rays and determine their path difference. A lens is necessary to achieve a focus on the screen. See Figure 5. If $\Gamma = \lambda/2$, the rays will interfere destructively at the screen. Additional pairs of points can be chosen similarly until the slit is full. Each pair will have rays at the same angle as those coming from points A and B, which will also interfere destructively. Thus, point P on the screen will be the location of a dark fringe. The location of the bright fringes is more complicated to determine. Suffice it to say that in between every two dark fringes there must be a light fringe.

Figure 3

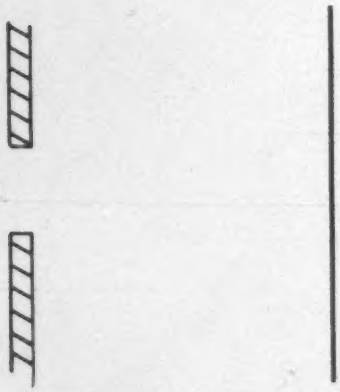
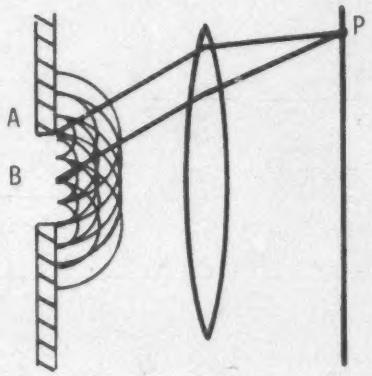


Figure 4



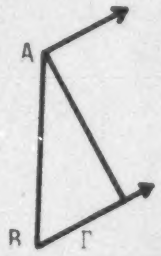
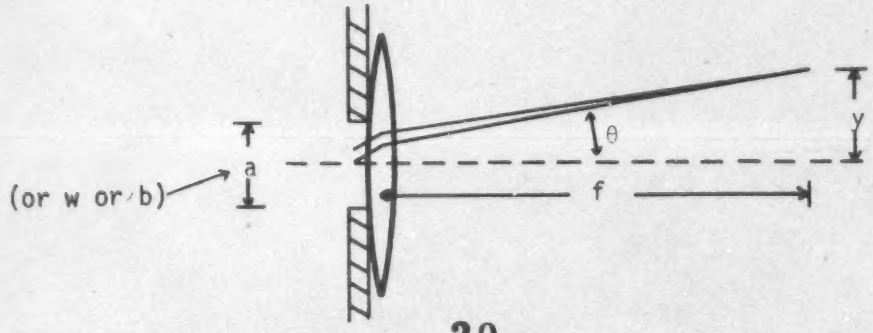


Figure 5

B(2). A thin lens with a 3.00-m focal length is placed directly to the right of a 0.60-mm-wide single slit. The slit is illuminated from the left with 500-nm-wavelength light. The intensity at the central maximum is 12.0×10^{-6} W/m^2. Assume Fraunhofer diffraction, and find the intensity 2.00×10^{-3} m to the side of the central maximum.

Figure 6



Solution

See Figure 6. The texts give the intensity equation for Fraunhofer diffraction as

$$I = I_0[(\sin^2 \alpha)/\alpha^2], \quad \alpha = (\pi a/\lambda) \sin \theta; \qquad \text{(HR)*}$$

$$I = I_0[(\sin^2 u)/u^2], \quad u = (\pi b/\lambda) \sin \theta; \qquad \text{(B)*}$$

$$I = I_0\left[\frac{\sin(\pi a/\lambda \sin \theta)}{\pi a/\lambda \sin \theta}\right]^2; \qquad \text{(SZ)*}$$

$$I = I_0\left[\frac{\sin^2 \phi/2}{\phi/2}\right]^2, \quad \phi = (2\pi w/\lambda) \sin \theta. \qquad \text{(WS)*}$$

Find θ: $\tan \theta = y/f = (2.00 \times 10^{-3}\ \text{m})/(3.00\ \text{m})$. For this small angle, $\tan \theta \simeq \theta$. Using the notation of Halliday and Resnick:

$$\alpha = \frac{\pi a}{\lambda}\left(\frac{y}{f}\right) = \frac{\pi(6.0 \times 10^{-4}\ \text{m})(2.00 \times 10^{-3}\ \text{m})}{(500 \times 10^{-9}\ \text{m})(3.00\ \text{m})} = 2.51.$$

Then

$$I = (12.0 \times 10^{-6}\ \text{J/m}^2\ \text{s})[\sin^2(144°)/(2.51)^2] = 6.58 \times 10^{-7}\ \text{J/m}^2\ \text{s}.$$

C(3). A counterfeiter photographs a bill prior to making his engraved plate. See Figure 7. His camera has a 2.00×10^{-2} m diameter lens, and he uses daylight (550×10^{-9} m). What is the farthest he can place his camera from the bill if he wants to be able to resolve details 1.00×10^{-4} m apart.

Solution

The limiting angle of resolution for a circular aperture is $\theta = 1.22\lambda/d$. The angle θ is related to the bill-camera distance by $y/x \simeq \tan \theta \simeq \theta$ for small angles, which you can anticipate here. See Figure 8. Thus, $y/x = 1.22\lambda/d$ and

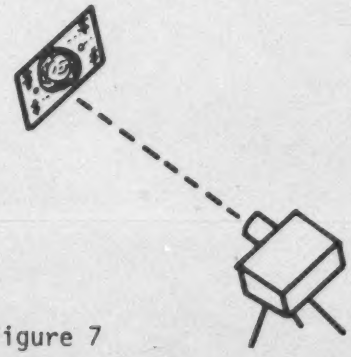


Figure 7

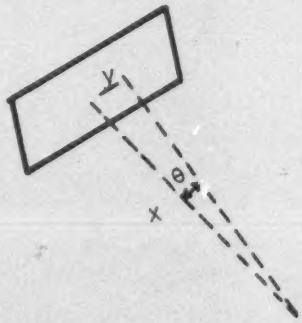


Figure 8

*HR = Halliday and Resnick. B = Bueche. SZ = Sears and Zemansky. WS = Weidner and Sells.

$$x = \frac{yd}{1.22\lambda} = \frac{(1.00 \times 10^{-4}\ \text{m})(2.00 \times 10^{-2}\ \text{m})}{(1.22)(550 \times 10^{-9}\ \text{m})} = 3.00\ \text{m}.$$

D(4). A diffraction grating has 5.5×10^5 lines/m. What is the highest order for which a 560×10^{-9} m green light can be observed? Assume normally incident plane waves.

Solution

The intense (principal) maxima are located by $m\lambda = d \sin\theta$. The maximum viewing angle is $\theta = \pi/2$. In this case, $m\lambda \leq d$ because m is an integer. Thus,

$$m \leq \frac{d}{\lambda} = \frac{1}{(5.5 \times 10^5/\text{m})(560 \times 10^{-9}\ \text{m})} = 3.2 \quad \text{and} \quad m = 3.$$

Problems

E(2). If the yellow light from a sodium arc ($\lambda = 589 \times 10^{-9}$ m) is used in a Franunhofer single-slit diffraction experiment, how wide must the slit be if the first minimum occurs at an angle of 6°? Would it be difficult to carry out this experiment?

F(2). A plane wave having wavelength 5.90×10^{-7} m falls on a slit of width 0.400×10^{-3} m. A converging lens, focal length of 0.70 m, is placed behind the slit and focuses the light on a screen. What is the distance on the screen from the center of the diffraction pattern to (a) the first minimum? (b) the second minimum?

G(3). Some persons who live in the Arctic reduce the amount of light entering their eyes by wearing opaque screens with slits cut in them as shown in Figure 9. What is the smallest width of the slits so that the "sunglasses" will not prevent resolution of objects 0.30 m apart and 500 m away? Assume sunlight with $\lambda = 6.00 \times 10^{-7}$ m and Fraunhofer diffraction.

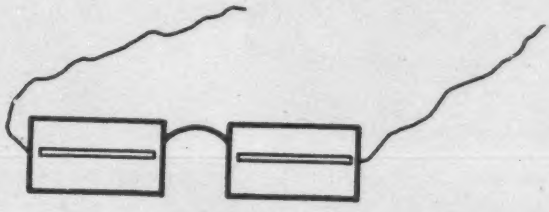


Figure 9

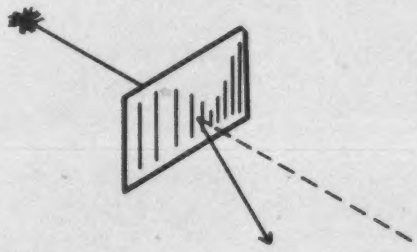


Figure 10

H(4). A student calibrates a diffraction grating using light from a helium-neon laser ($\lambda = 6.328 \times 10^{-7}$ m) and Fraunhofer conditions. See Figure 10. The first-order principal (or most intense) maximum occurs at 38°.

(a) What is the spacing between the rulings on the grating? (This is what the calibration accomplishes.)

(b) At what angle is the second-order principal maximum?

I(4). A diffraction grating 0.0200 m wide has 6000 rulings. At what angles will the principal (or most intense) maxima occur if the incident radiation has a wavelength of 5.89×10^{-7} m. Assume Fraunhofer conditions.

Solutions

E(2). 5.6×10^{-6} m. Yes, if you had to make your own apparatus. The slit is very narrow.

F(2). (a) 1.03×10^{-3} m. (b) 2.06×10^{-3} m.

G(3). 1.00×10^{-3} m. H(4). (a) 1.03×10^{-6} m. (b) It does not occur.

I(4). First order: 10°. Second order: 21°. Third order: 32°. Fourth order: 45°. Fifth order: 62°.

PRACTICE TEST

1. Use Huygens' principle to explain how light through a single slit can produce interference fringes.

2. (a) State the optical conditions necessary for Fraunhofer diffraction.
(b) What is the distance between the central maximum and the third minimum of a Fraunhofer single-slit diffraction pattern on a screen 0.40 m away from the slit? The light has a wavelength of 5.50×10^{-7} m, and the slit is 2.50×10^{-3} m wide.

3. (a) Use the Fraunhofer single-slit diffraction pattern intensity graph to describe the conditions for which two objects are just resolved.
(b) A telescope is used to observe two distant point sources 0.50 m apart. The light used has a wavelength of 5.00×10^{-7} m, and the objective mirror of the telescope is covered with a screen having a rectangular slit of width 1.00×10^{-3} m. What is the maximum distance at which the two sources may be distinguished as two?

4. What is the longest wavelength that can be observed in the third order for a diffraction grating having 1.00×10^{6} lines/m?

Practice Test Answers

2. (b) 2.60×10^{-4} m. 3. (b) 1.00×10^{3} m. 4. 3.3×10^{-7} m.

DIFFRACTION

Date _____

Mastery Test Form A

pass recycle

1 2 3 4

Name _____ Tutor _____

1. A plane wavefront advances from the left toward the single slit shown in Figure 1.

   (a) Use Huygens' principle to explain how the wave can go through point P.

   (b) Use Huygens' principle to explain briefly how there might be a light or dark fringe on a screen placed at point P.

2. (a) Why is a lens necessary to focus the fringes resulting from Fraunhofer diffraction?

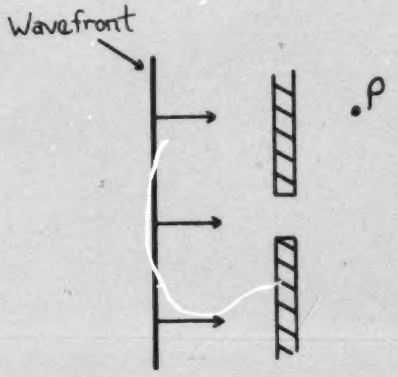
   (b) Light of wavelength 4.00×10^{-7} m is used in a Fraunhofer single-slit experiment. The slitwidth is 4.5×10^{-4} m. What is the ratio of the intensity at 2.00° to the intensity at the central maximum?

3. (a) Briefly describe the conditions on the separation of their diffraction patterns for two objects that are just resolved.

   (b) An astronaut lands on the Moon and sets off two bright, yellow flares ($\lambda = 590 \times 10^{-9}$ m) 10.0 m apart. The Moon is 3.8×10^{8} m from Earth. Could the 200-in. (5.1-m) telescope on Mt. Palomar resolve the flares? Explain.

4. A diffraction grating having 10 000 lines per centimeter produces a first-order principal (or most intense) maximum at $\theta = 28°$. What is the wavelength of the light used?

Figure 1



DIFFRACTION Date _____

Mastery Test Form B

pass recycle

1 2 3 4

Name _____ Tutor _____

1. A plane wavefront advances from the left toward the single slit shown in Figure 1.
   (a) Use Huygens' principle to explain how the wave can go through point P.
   (b) Use Huygens' principle to explain briefly how there might be a light or dark fringe on a screen placed at point P.

2. (a) What are the optical conditions necessary to produce Fraunhofer diffraction through a single slit?
   (b) You are given the task of measuring the width of a very narrow slit and you decide to use Fraunhofer diffraction. Your apparatus is shown in Figure 2. A 1.50-m-focal-length lens is placed against the slit. You measure the distance between the second- and third-order dark fringes to be 5.0×10^{-2} m. What is the width of the slit? ($\lambda = 475 \times 10^{-9}$ m.)

3. (a) The intensity diffraction pattern for a small object is shown in Figure 3. A second similar small object is placed near the first, and it can just be determined that two objects are present. Sketch the intensity diffraction pattern for the second object in Figure 3.
   (b) An approaching car with headlights separated by 1.40 m is viewed through a single slit 1.00×10^{-4} m wide. How close must the car be to the observer before she can distinguish that there are two headlights? ($\lambda = 7.00 \times 10^{-7}$ m.)

4. You are given a diffraction grating with 6000 lines per centimeter.
   (a) What is the highest order you can observe for a principal (most intense) maximum for the 632.8×10^{-9} m wavelength helium-neon laser light?
   (b) At what angle does this order occur.

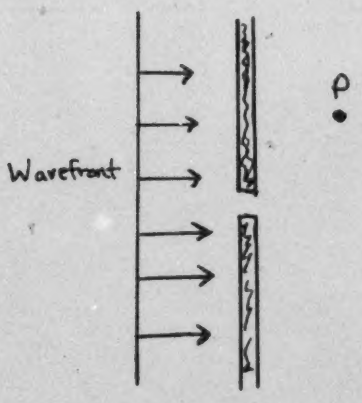
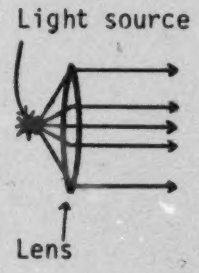


Figure 1



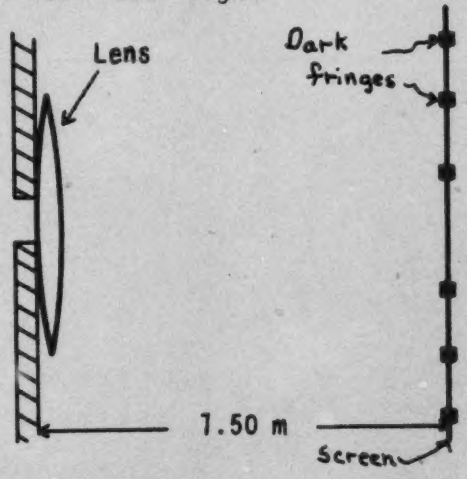


Figure 2

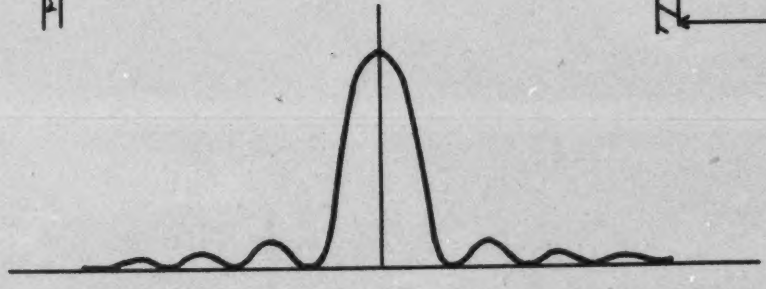


Figure 3

DIFFRACTION Date _____

Mastery Test Form C

pass recycle

1 2 3 4

Name _____ Tutor _____

1. A plane wavefront advances toward the single slit from the left in Figure 1.
   (a) Use Huygens' principle to explain how the wave can go through point P.
   (b) Use Huygens' principle to explain briefly how there might be a light or dark fringe on a screen placed at point P.

2. (a) In the apparatus shown in Figure 2, where f_1 and f_2 are the focal lengths of Lenses 1 and 2, respectively, will the diffraction of light through the single slit be Fraunhofer diffraction? Briefly explain why.
   (b) If the light source emits two wavelengths, 500 and 750 × 10^{-9} m, show that the second-order dark fringe for one will coincide on the screen with the third-order dark fringe of the other.

3. (a) A small object is placed near the similar small object whose intensity diffraction pattern is shown in Figure 3, and it can just be determined that two objects are present. Sketch the intensity diffraction pattern for the second object on Figure 3.
   (b) A circular radar antenna (λ = 0.250 m) is designed to resolve two airplanes 1000 m apart and 200 km away from the antenna. What is the minimum diameter of the antenna dish?

4. You are given a diffraction grating and told to determine the number of lines per meter. Using the green line of mercury (546 × 10^{-9} m), you find that the angle between the central and the first principal (most intense) maxima is 30°.

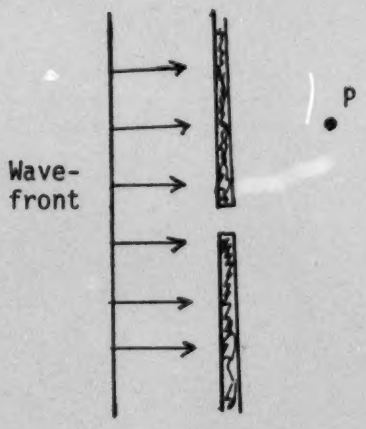


Figure 1

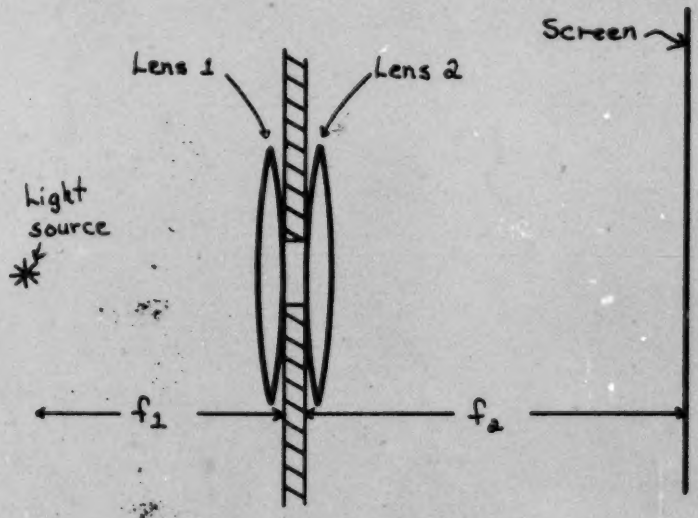


Figure 2

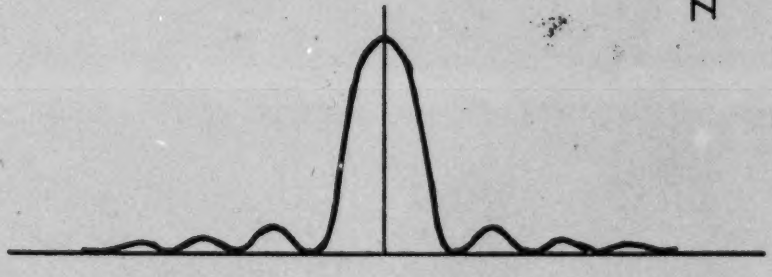


Figure 3

MASTERY TEST GRADING KEY - Form A

1. Solution: See Figure 18. Each point in the slit is a source of secondary spherical waves. Rays from any of these waves can go to point P.
(b) If the spherical waves that go through point P have pathlength differences such that their superposition causes complete destructive or constructive interference, there will be fringes at P.

2. Solution: (a) Fraunhofer diffraction is characterized by plane wavefronts (parallel rays) on both sides of the obstacle or window. A lens is necessary to focus parallel rays.
(b) Use $I = I_0[(\sin^2 \alpha)/\alpha^2]$, where $\alpha = \pi a/\lambda \sin \theta$.

Solving for α, we find

$$\alpha = \pi(4.5 \times 10^{-4}\ \text{m})/(4.00 \times 10^{-7}\ \text{m}) \sin(2.00°).$$

Use the small-angle approximation $\sin \theta \simeq \theta$ to get $\sin 2.00° = \pi/90$. Then

$$\alpha = \pi^2(4.5 \times 10^{-4}\ \text{m})/(4.00 \times 10^{-7}\ \text{m})(90) = 123\ \text{rad}.$$

Now

$$\frac{I}{I_0} = \frac{\sin^2 \alpha}{\alpha^2} = \frac{\sin^2(211°)}{(123)^2} = \frac{\sin^2(31°)}{(123)^2} = 1.75 \times 10^{-5}.$$

3. Solution: (a) The central maximum of one object's diffraction pattern is at the first minimum of diffraction pattern of the other object.
(b) Use $\sin \theta = 1.22\ \lambda/a$ and anticipate that θ will be very small so that $\sin \theta \simeq \tan \theta$. See Figure 19. $\tan \theta = y/x$, and thus, $y/x \geq \theta$ is the condition for resolution of the flares.

$$\frac{y}{x} = (10.0\ \text{m})/(3.8 \times 10^{8}\ \text{m}) = 2.6 \times 10^{-8}, \qquad \theta = 1.22[(590 \times 10^{-9}\ \text{m})/5.1\ \text{m}]$$
$$= 14.1 \times 10^{-8}.$$

Thus the flares are not resolved.

4. Solution: Use $m\lambda = d \sin \theta$.

$$\lambda = (d \sin \theta)/m = (1.00 \times 10^{-6}\ \text{m})(0.469)/1 = 469 \times 10^{9}\ \text{m}.$$

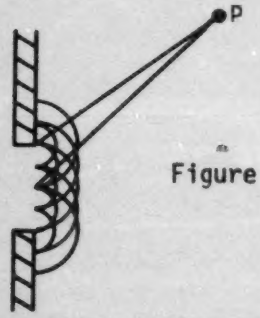


Figure 18

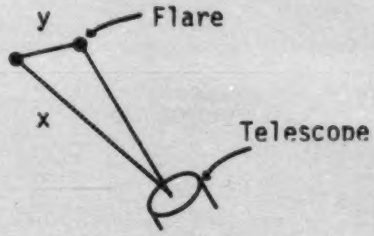


Figure 19

MASTERY TEST GRADING KEY - Form B

1. Solution: (a) See Figure 20. Each point in the slit is a source of secondary spherical waves. Rays from any of these waves can go to point P.
(b) See Figure 20. If the spherical waves that go through point P have path-length differences such that their superposition causes complete destructive interference, there will be fringes at P.

2. Solution: (a) The wave on the upstream side of the slit must have plane wavefronts. The analysis for interference on the downstream side of the slit is done with parallel rays (also plane wavefronts).
(b) See Figure 21. Using $m\lambda = a \sin\theta$, and the small-angle approximation $\theta \simeq \sin\theta \simeq \tan\theta$: $m\lambda = ay/x$, and for the second and third dark fringes,

$$2\lambda = ay_2/x, \quad 3\lambda = ay_3/x, \quad y_3 - y_2 = (3 - 2)\lambda x/a,$$

$$a = \frac{\lambda x}{y_3 - y_2} = \frac{(475 \times 10^{-9}\ \text{m})(1.50\ \text{m})}{5.0 \times 10^{-2}\ \text{m}} = 1.43 \times 10^{-4}\ \text{m}.$$

3. Solution: (a) See Figure 22. The central maximum of one pattern must be over the first minimum of the other.
(b) Use $\sin\theta = \lambda/d = (7.00 \times 10^{-7}\ \text{m})/(1.00 \times 10^{-4}\ \text{m}) = 7.00 \times 10^{-3}$, and the small-angle approximation $\sin\theta \simeq \tan\theta$ is justified. See Figure 23, where $\tan\theta = y/x$. Thus:

$$x = \frac{y}{7.00 \times 10^{-3}} = \frac{1.40\ \text{m}}{7.00 \times 10^{-3}} = 200\ \text{m}.$$

4. Solution: (a) Use $m\lambda = d \sin\theta$ and, since $|\sin\theta| \leq 1$,

$$m \leq \frac{d}{\lambda} = \frac{1}{(6 \times 10^5/\text{m})(632.8 \times 10^{-9}\ \text{m})} = 2.6.$$

Thus $m_{max} = 2$.
(b) $\sin\theta = m\lambda/d = 2/2.60 = 0.77$. $\theta = 51°$.

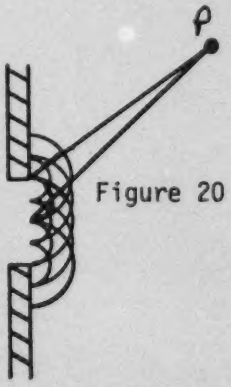


Figure 20

Figure 21

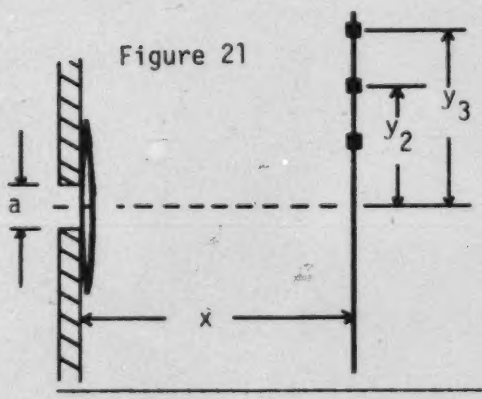
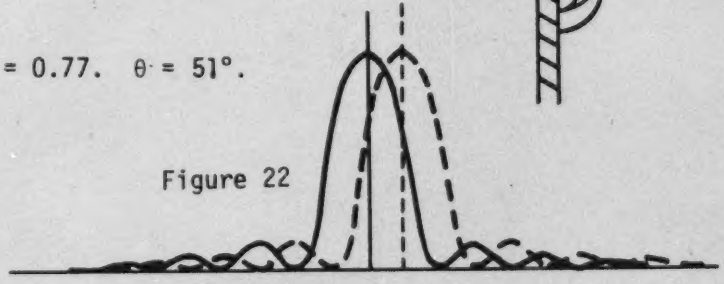


Figure 22



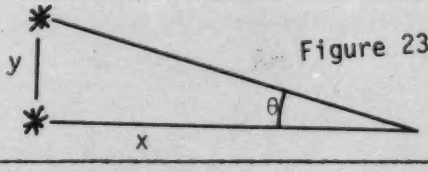


Figure 23

MASTERY TEST GRADING KEY - Form C

1. Solution: See Figure 24. Each point in the slit is a source of secondary spherical waves. Rays from any of these waves can go to point P.

   (b) If the spherical waves that go through point P have pathlength differences such that their superposition causes complete destructive or constructive interference, there will be fringes at P.

2. Solution: (a) Yes. Lens 1 causes parallel-ray (plane wavefront) light to shine on the slit; and lens 2 focuses only parallel rays onto the screen.

   (b) Use $m\lambda = a \sin\theta$. When the two orders coincide they will have the same θ. The slitwidth a is the same for both wavelengths, thus $m_1\lambda_1 = m_2\lambda_2$. $2\lambda_1 = 3\lambda_2$ and $\lambda_1/\lambda_2 = 3/2$, which is satisfied for $\lambda_1 = 750 \times 10^{-9}$ m and $\lambda_2 = 500 \times 10^{-9}$ m.

3. (a) See Figure 25. The central maximum of one pattern must be over the first minimum of the other.

   (b) Use $\sin\theta = 1.22\lambda/d$. Since the planes are two hundred times farther away than the distance between them you can use the small-angle approximation $\sin\theta \approx \tan\theta$. See Figure 26. $\tan\theta = y/x = 1.22\lambda/d$ and

$$d = \frac{1.22\lambda x}{y} = \frac{(1.22)(0.250\text{ m})(200 \times 10^3\text{ m})}{1000\text{ m}} = 61\text{ m}.$$

4. Solution: Use $m\lambda = d \sin\theta$. d is the distance between the lines. We want 1/d. Thus

$$\frac{1}{d} = \frac{\sin\theta}{m} = \frac{0.50}{1(546 \times 10^{-9}\text{ m})} = 9.1 \times 10^5/\text{m}.$$

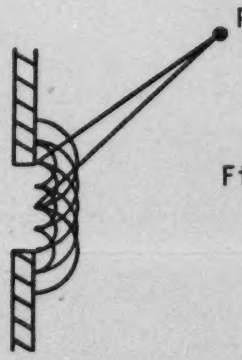


Figure 24

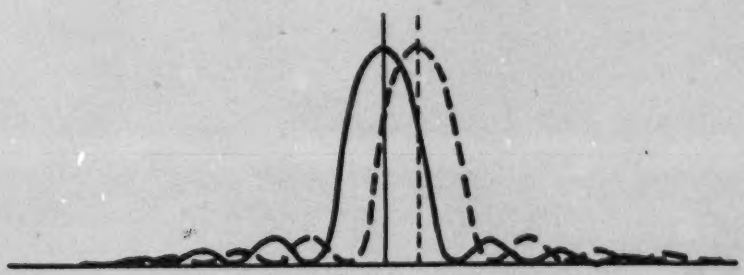


Figure 25

ALTERNATING-CURRENT CIRCUITS

INTRODUCTION

The electric clock on the wall, radio and television, the incredibly rapid handling of information by computers, and the transmission of signals by our own nerves are among countless devices and effects that depend on circuits in which currents or voltages vary with time. Alternating-current (ac) circuits, in which charges oscillate back and forth in a wire in such a way that the average current is zero, are among the simpler time-varying circuits. In this module you will study the behavior of simple ac circuits containing resistors, inductors, and capacitors.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Convert degrees to radians and radians to degrees (needed for Objective 1 of this module)	Trigonometry Review
*Differentiate and integrate sine and cosine functions (needed for Objectives 1 and 2 of this module)	Calculus Review
*Relate emf, current, and resistance in dc circuits (needed for Objectives 1 through 3 of this module)	Direct-Current Circuits Module
*Relate capacitance, charge, and potential difference (needed for Objectives 1 through 3 of this module)	Capacitors Module
*Relate inductance, current, and emf (needed for Objectives 1 through 3 of this module)	Inductance Module
*Calculate energy in the electric field of a capacitor (needed for Objective 4 of this module)	Capacitors Module
*Calculate energy in the magnetic field of an inductor (needed for Objective 4 of this module)	Inductance Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Rotating vectors - Illustrate phase relations among currents and potential differences in different portions of a given ac circuit, using vector and/or graphical techniques.

2. Alternating-current analysis - Solve for unknown quantities in an ac circuit, using vector techniques, given a suitable set of currents, potential differences, emfs, resistances, capacitances, inductances, and/or frequencies.

3. Averages - Calculate root-mean-square (rms) potential differences, currents, and/or average power in a given ac circuit.

4. Energy - Describe the energy flow in a given ac circuit.

TEXT: Frederick J. Bueche, Introduction to Physics for Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

You have already studied simple examples in which currents or potential differences varied with time. For Objective 1, it will be useful to review quickly Sections 21.12 in Chapter 21 and Section 25.4 in Chapter 25. Then study Section 27.1 in Chapter 27 for Objective 2. Usually angular frequency, $\omega = 2\pi f$ (radians per second), is used to avoid carrying factors of 2π along in all the equations.

Study General Comments 1 through 3 and Chapter 27, Sections 27.2 through 27.4 for Objective 3. Keep in mind the text's convention on notation: v, i are time-varying quantities; v_0, i_0 are the maximum values, or amplitudes of time-varying quantities; and V, I are rms quantities. To make the text on p. 524 consistent with Figure 27.3, change "C" and "D" to "A" and "B," respectively, in the second paragraph and in the footnote.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 27)	(Chap. 27)
1	Sec. 27.4, General Comments 3, 5	A	Illus.[a] 27.5, 27.6, 27.7	D	7, 8, 14	9, 10, 15, 16, 19
2	Secs. 27.2, 27.3, 27.5, General Comments 1, 5	B	Illus. 27.2, 27.3, 27.4	E	3, 5, 7, 8, 14	Quest.[a] 9 to 13, Probs. 4, 6, 9, 10 to 13, 15, 16, 19
3	Sec. 27.1, General Comments 2, 5	C		F		Quest. 5, Probs. 1, 2, 9, 10
4	Sec. 27.6, General Comment 4	B, C		E, F		Quest. 3, 4

[a]Illus. = Illustration(s). Quest. = Question(s).

The vector treatment used to represent rms potential differences and currents (e.g., Fig. 27.8) can be extended to give instantaneous values, v(t) and i(t). Suppose the potential difference between two points in a circuit is given by the expression

$$v_1 = v_{10} \sin \omega t = \sqrt{2}V \sin \omega t,$$

where $V = [v(t)]_{rms}$. Let us draw a vector representing the amplitude, $v_{10} = \sqrt{2}V$. If the vector is horizontal at t = 0 and is allowed to rotate counterclockwise with angular velocity ω, it will look like the sketches in Figure B1 at the stated times. The instantaneous value of v(t) at time t is then simply the vertical component of the rotating vector.

If a potential difference is the sum of two potential differences, with a difference in phase, as in Figure 27.8, the same analysis can be applied. The entire vector triangle rotates with angular velocity ω. Thus, if $i = i_0 \sin \omega t$ as in Figure 27.8, the vector triangle looks, at different times, like Figure B2. In summary, to find the instantaneous potential difference or current at any instant t, algebraically add the component instantaneous values. To find the amplitude or the rms potential difference, add the component values vectorially. In comparing the triangles drawn here with those in Figure 27.8, notice that Bueche has drawn the triangle for rms values, but we have plotted maximum values. This does not affect the geometry in any way, since we multiply the length of each rms quantity by the same factor $\sqrt{2}$ to get the amplitude. Solve Problems A and B. Compare your solutions with those in the study guide. Then solve Problems D and E, and Problems 3, 5, 7, 8 and 14 in Chapter 27.

Study Sections 27.5 and 27.6. Resonance is an important special case of the general RLC circuit. If you can handle the general case, resonance is easy! In Section 27.6 the author suddenly changes his notation. To avoid confusion, add the subscript "av" to the left-hand side of Eq. (27.14b) and the unnumbered equation following it. Read General Comments 4 and 5. Solve Problem C and compare your answer to that in the study guide. Then solve Problems F. Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

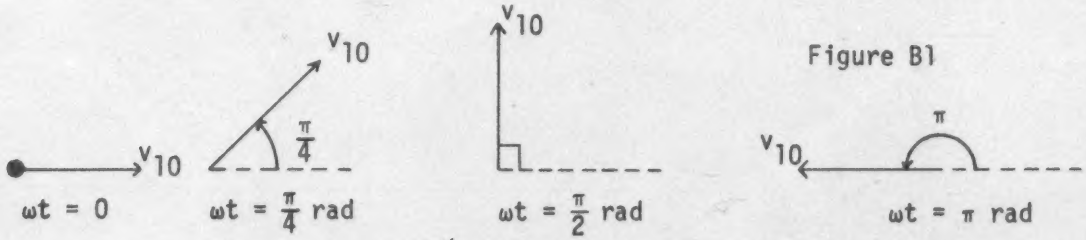


Figure B1

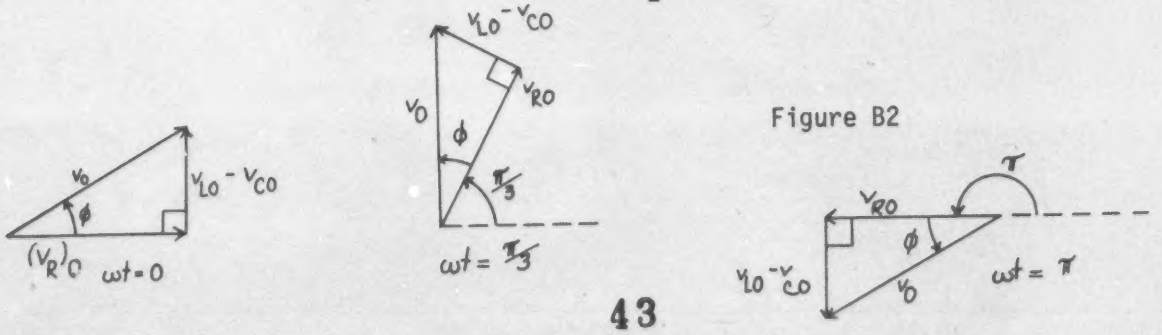


Figure B2

STUDY GUIDE: Alternating-Current Circuits 3(HR 1)

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Your text does not treat the subject of ac circuits. If you have available any of the books listed below, your instructor can supply you with a study guide for this module:

Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition.

Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition.

Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2.

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

You have already studied simple examples in which currents or potential differences varied with time. It will be useful to review quickly Chapter 29, Section 29-7, Chapter 33, Sections 33-11, 33-12, and 33-13. Study General Comments 1 and 2. Then work Problems 35-1(a), (b), (c), and 35-3(a), (b), (c).

Study Sections 35-1 and 35-2. Notice the conventions used by your text: lower-case letters imply instantaneous values, upper-case letters imply amplitudes, or maximum values. The use of rotors was first described in Section 11-4. Review that section if the use of rotor diagrams seems unfamiliar. Rotors are also called rotating vectors or phasors. Solve Problem A and compare your solution with that given. Then work Problem D. The word "voltage," derived from the volt unit, is commonly used for either potential difference or emf. Its useage is so common that you will have to learn to live with it.

Study Section 35-3. Impress on your mind the last paragraph on p. 507. Kirchhoff's rules for circuit analysis depend ultimately on two of the great conservation laws of nature: the conservation of charge and the conservation of energy. We sometimes get so accustomed to applying "rules" that we forget what physics lies behind them.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 11-4, 35-3, General Comments 3, 5	A	D	35-6 to 35-9, 35-11, 35-13	35-10, 35-12, 35-14, 35-15, 35-16, 35-18, 35-19, 35-20
2	Secs. 35-1, 35-2, 35-3, 35-6, General Comments 1, 5	B	E	35-1(a), (b), (c), 35-3(a), (b), (c)	35-10, 35-12, 35-14 to 35-20
3	Secs. 35-4, 35-5, General Comments 2, 5	C	F		35-12, 35-14 to 35-17, 35-19
4	General Comment 4	B, C	E, F		

Study Sections 35-4 and 35-5. Study General Comment 3. Then solve Problem B and compare your solution with the one provided. Solve Problem E and Problems 35-6 to 35-9, 35-11, and 35-13. Study Section 35-6. Resonance is an important special case of the general RLC circuit. If you can handle the general case, resonance is easy! Study General Comment 5. Solve Problem C and compare your solution with the one in the study guide. Then solve Problem F.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

You have already studied simple examples in which currents or potential differences varied with time. It will be useful to review quickly Chapter 27, Section 27-7, Chapter 32, Section 32-2, and Chapter 34, Section 34-1. If you have not done so, correct Eq. (34-7); the first term on the right-hand side should read

$$Q_m^2(\cos \omega t)^2/2C.$$

Study carefully Section 34-2. Keep in mind the notation used by your text for instantaneous, rms, and maximum values. Rotating vectors are sometimes abbreviated rotors. Another common term for them is phasors. By convention the vector rotates counterclockwise, thus ωt is the angle measured counterclockwise from the position at time $t = 0$. The important point when adding rotating vectors is that instantaneous values add algebraically, amplitudes (maximum values) or rms values add like vectors. Solve Problem A in this study guide, and compare your solution with the one provided. Then solve Problem D. Study General Comments 1 to 3 in the study guide. Then study Sections 34-3 through 34-5. Note the common, but imprecise, use of the word "voltage" to refer either to potential difference or emf. Equation (34-16) states that the average power in the RC circuit is $P = VI$, where V and I are rms values of v and i. This is the average power delivered to the circuit by the source of emf. Where does that power come from? Does the generator create the energy? What happens to the energy when it is delivered to the circuit? Solve Problem 34-4.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 34-2, 34-6, General Comments 3, 5	A	Ex.[a] 34-1	D	34-4, 34-8, 34-10 to 34-12	34-3, 34-6, 34-7, 34-9, 34-13, 34-14
2	Secs. 34-2 to 34-6, General Comments 1, 5	B	Ex. 34-1 to 34-3	E	34-8, 34-10 to 34-12	34-3, 34-6, 34-7, 34-13, 34-14
3	Secs. 34-2 to 34-5, General Comments 2, 5	C	Ex. 34-2, 34-3	F	34-8, 34-11, 34-12	34-9, 34-13
4	General Comment 4	B, C		E, F		34-3

[a]Ex. = Example(s).

Study Section 34-6, including especially Example 34-3. Notice that the circuit analysis depends ultimately on two of the great conservation laws; conservation of charge and conservation of energy. We sometimes get so accustomed to applying "rules" that we forget the physics that lies behind them. Study Section 34-7. The resonant circuit is, of course, just a special case of the RLC series circuit. Solve Problems B and E. Then solve Problems 34-8, 34-10, 34-11, and 34-12. In Problem 34-11, draw a graph of v_C, v_R, and $\mathcal{E}$ as functions of time. Study General Comment 5, and solve Problems C and F.

Take the Practice Test, and work some Additional Problems if necessary, before attempting a Mastery Test.

GENERAL COMMENTS

1. Signs

In writing the differential equation representing Kirchhoff's loop equation for a circuit, you must be precise and definite in your choice of signs. For example, suppose we have an RC circuit as in Figure 1. We can choose any convention we wish for the signs, but we must make a choice and stick with it. In this case, a common choice is to say that the current is positive clockwise when the top of the source of emf is at a positive potential with respect to the bottom. We still must choose a sign for the charge on C. Let us choose the bottom plate to have positive charge. (That is a reasonable choice. If $\mathcal{E}$ were missing, +q on the bottom plate would provide a clockwise current.) Now the circuit diagram looks like Figure 2. Kirchhoff's rule tells us to set the emf equal to the potential difference between points a and b:

$$iR - q/C = \mathcal{E}(t).$$

Differentiating with respect to time gives us

$$R\left(\frac{di}{dt}\right) - \left(\frac{1}{C}\right)\left(\frac{dq}{dt}\right) = \frac{d\mathcal{E}(t)}{dt} .$$

Now the temptation is to set dq/dt = i. However, with the signs chosen here, positive current means that the magnitude of the charge stored in each plate of C will decrease. We therefore must set

$$dq/dt = -i.$$

The circuit equation then becomes

$$R\left(\frac{di}{dt}\right) + \left(\frac{1}{C}\right)i = \frac{d\mathcal{E}(t)}{dt} ,$$

and the work of solving it can begin.

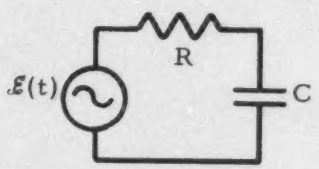


Figure 1

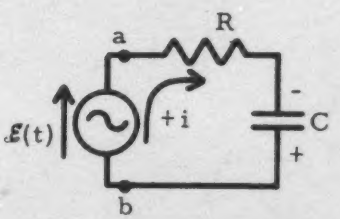


Figure 2

If, on the other hand, we had chosen the reverse convention for the sign of the charge on C, a positive current would cause an increase in the charge stored on the plate. The equation would then read

$$iR + q/C = \mathcal{E}(t) \quad \text{with the condition } dq/dt = i.$$

This gives us, after differentiation,

$$R\left(\frac{di}{dt}\right) + \frac{i}{C} = \frac{d\mathcal{E}(t)}{dt} .$$

We get the same equation, as we must, since it describes the same physical situation. But we get it only if we are consistent in dealing with signs.

In a circuit with an inductor, if the current increases in the direction of positive current ($di/dt > 0$), we know there will be an opposing emf, trying to prevent the increase in current:

$$\mathcal{E}_L = -L(di/dt).$$

See Figure 3. This means that at the time shown in Figure 3, with positive di/dt, the induced emf in the inductor will tend to drive an opposing current counter-clockwise. The net emf in going around the circuit is $\mathcal{E} - L(di/dt)$. (See the redrawn version of the same circuit in Figure 4.)

$$\mathcal{E}(t) - L(di/dt) = iR; \quad \mathcal{E}(t) = L(di/dt) + iR.$$

Try to reach the point where you can analyze a circuit physically to be sure you get the signs correct. Rules memorized without physical understanding are fallible and readily forgotten.

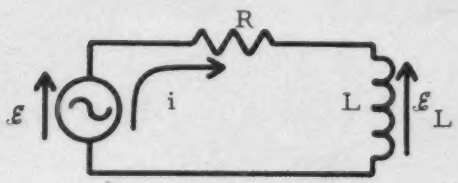


Figure 3

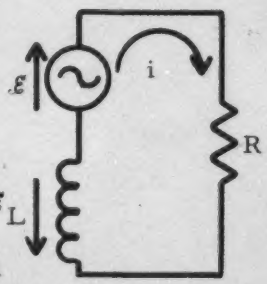


Figure 4

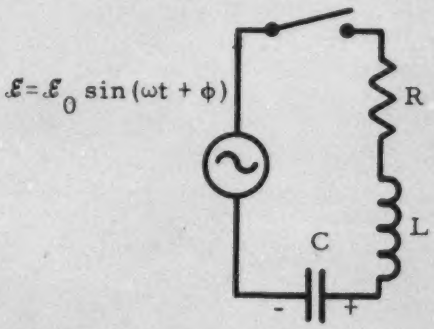


Figure 5

2. Approximations

Consider an RLC circuit, with an ac source as in Figure 5. The phase ϕ in the emf is to allow for the fact that the initial phase of the emf may depend on the particular conditions. (The emf might not be zero at the instant you turn on the switch, for example.) The loop equation for the circuit, assuming the signs given, is

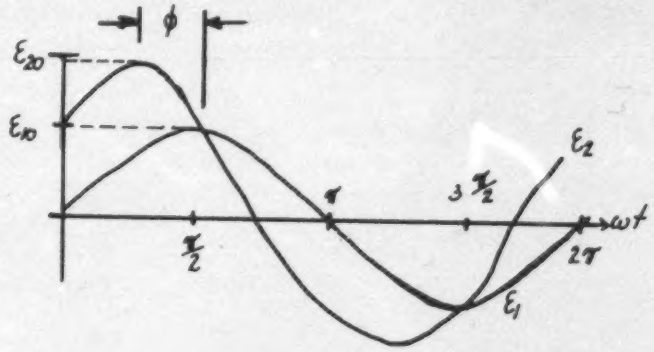
$$iR + q/C = \mathcal{E}_0 \sin(\omega t + \phi) - L(di/dt).$$

Differentiating, rearranging, and using the fact that $i = dq/dt$ gives us

$$L\left(\frac{d^2i}{dt^2}\right) + R\left(\frac{di}{dt}\right) + \left(\frac{1}{C}\right)i = \frac{d\mathcal{E}(t)}{dt} = \omega\mathcal{E}_0 \cos(\omega t + \phi).$$

This differential equation, along with the initial conditions, determines the complete, exact behavior of the current for all time $t > 0$, up to the time when the switch is again opened. There are mathematical techniques for solving this equation. The solution will give you the current, including transient effects. However, since we do not wish to plunge into that much mathematical detail, we can learn a great deal about the circuit by making some assumptions and approximations. In so doing, we lose some of the detailed information contained in the exact solution of the differential equation.

Figure 6



One important assumption we make throughout this module is that we are most interested in the steady-state solution; that is, the solution after the currents and potential differences have had time to reach their maximum final amplitudes. That means we shall not learn how the current builds up from zero to its final value.

Another implicit assumption in the treatment used here is that the current has the same frequency as the applied emf. That is certainly true, but is not proved here.

Because the frequencies are typically so large that we can ignore the details of the oscillation of V or i, we frequently are interested only in average or rms values. That is what you read on an ac voltmeter or ammeter. Of course, you are sacrificing a lot of information that would be contained in the exact solution of the differential equation.

3. Phase

When two oscillations of the same frequency are superimposed, the result depends on the relative timing of the oscillations. At one extreme, the maxima of both oscillations occur at the same instant, and the oscillations add constructively. At the other extreme, the maximum of one oscillation coincides with the minimum of the other, and the oscillations subtract. There are, of course, infinitely many cases in between the two extremes.

The relative timing of the oscillations is determined by the relative phase of the two. Mathematically, the relative timing can be expressed by equations such as

$$\mathcal{E}_1 = \mathcal{E}_{10} \sin \omega t \quad \text{and} \quad \mathcal{E}_2 = \mathcal{E}_{20} \sin(\omega t + \phi),$$

where ϕ is the phase angle of $\mathcal{E}_2$ with respect to $\mathcal{E}_1$. A plot of these two expressions, for $\phi = \pi/4$ rad = 45° looks like Figure 6. Notice that a crest of $\mathcal{E}_2$ occurs at an angle $\omega t = \pi/4$ earlier than a crest of $\mathcal{E}_1$. The difference between leading and lagging is sometimes confused because, in our right-handed society, $\mathcal{E}_2$ appears to the left of $\mathcal{E}_1$, and therefore looks as if it's "behind" $\mathcal{E}_1$. Remember, the abscissa is proportional to the time, not the position. If one wave precedes the other in time, it leads!

If you simply try to memorize the fact that the potential difference leads the current for an inductor, and the current leads the potential difference in a capacitor, that knowledge probably will not stay with you very long. Even if you resort to mnemonic trickery, it will not help you learn physics. The correct way to learn which quantity leads which is to reason physically.

An inductor is a coil of wire. A current in the wire will produce a magnetic field through the coil, and therefore a magnetic flux. If the current changes, the flux will change, and Faraday's law, $\mathcal{E} = -d\Phi/dt = -L(di/dt)$, tells us that the changing flux produces an induced emf to oppose the change. Because of the opposing emf, it is hard to force a change in current in the wire of an inductor. In fact, if you try to abruptly change the current, the emf approaches infinity, since it is proportional to di/dt. Because the induced emf always opposes a current change, the current change cannot "keep up with" an imposed change in emf. The change in current always lags behind a change in the imposed emf.

To change the potential difference of a capacitor, we must transport charge from one of the plates to the other. To change the potential difference abruptly, we must transport a lot of charge in a very short time. In the limit of very short times, we approach an infinite derivative,

$$dV/dt = 1/C(dq/dt) = (1/C)i.$$

Since we cannot get an infinite current, we cannot change the potential across a capacitor in an infinitesimal time. The potential change cannot "keep up with" the current; the potential lags behind the current.

4. Energy

In charging up a capacitor, the source of emf performs work on the charges, which increases the energy stored in the electric field of the capacitor. We know that a capacitor C, charged to a potential difference V, will have energy

$$E = (1/2)CV^2$$

stored in the electric field. Similarly, an inductor with a current I has stored in its magnetic field the energy

$$E = (1/2)LI^2.$$

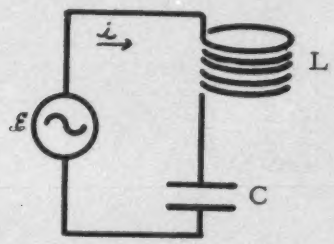


Figure 7

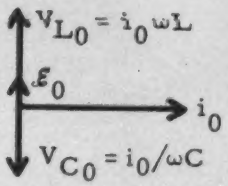


Figure 8

In an ac circuit containing capacitors and inductors, and possible resistors, there is clearly some kind of energy exchange going on between the electric and magnetic fields, with the source of emf available to do work or have work done on it. To try to understand what is going on, consider a simple LC series circuit, as in Figure 7, in which we assume that the inductor has zero resistance. We construct a vector diagram for the circuit in Figure 8, referring all vectors to the current, since all elements of the circuit have the current in common. The net emf $\mathcal{E}$ is the difference between V_L and V_C, since those two vectors are 180° out of phase. Let us assume that $\omega L > 1/(\omega C)$, so that $\mathcal{E}$ is in phase with V_L. We can now plot the emf, the current, and the potential differences as functions of the time - see Figure 9. In Figure 10 are shown the configurations of fields at the times t_1, t_2, t_3, and t_4 that are indicated on the graphs of Figure 9. These configurations are easy to figure out, if you recall that the magnetic field is proportional to the current, and the electric field is proportional to the potential difference across the capacitor. We see that between t_1 and t_2 the magnetic field is collapsing, and the electric field is building up. The energy in the circuit oscillates between the inductor and the capacitor, as the fields build and collapse in the two elements.

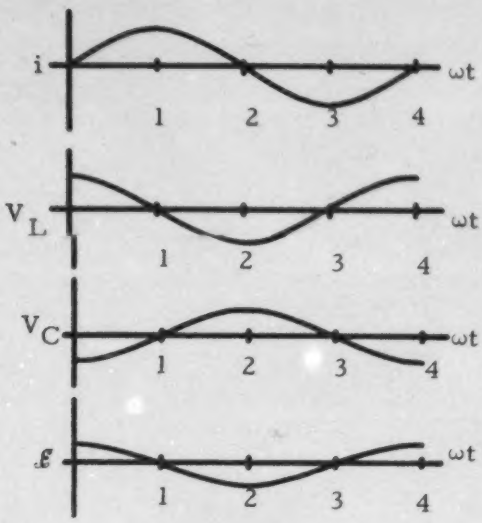


Figure 9

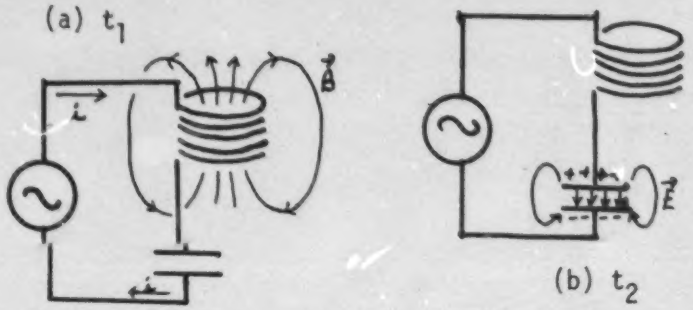
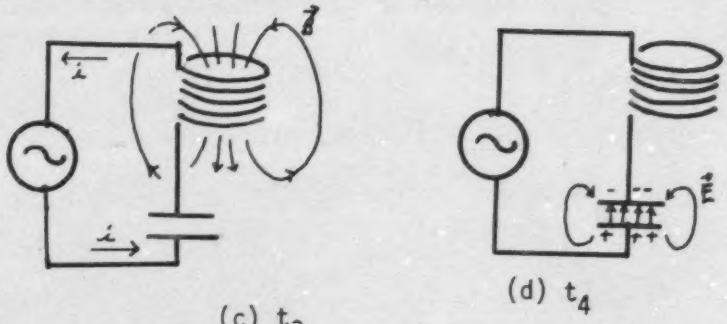


Figure 10



5. More Complicated Circuits

You might by now have the impression that all ac circuits must be hooked up in series. That is not true, and this comment will emphasize a general approach to all ac circuit problems.

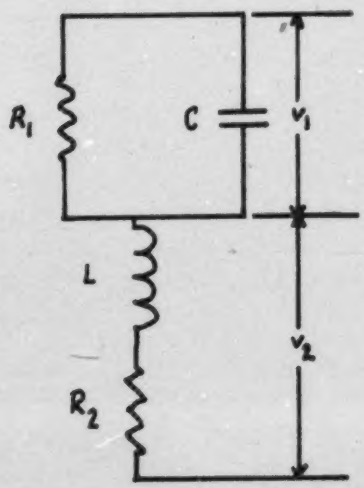


Figure 11

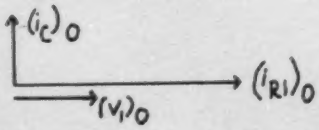


Figure 12

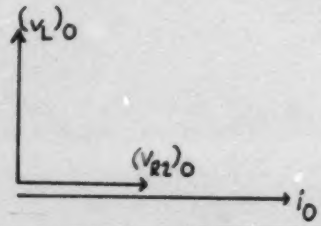


Figure 13

The key is to break a complicated circuit into sections in which all elements have a potential difference or a current in common, and to draw a vector diagram for each section, referring each vector in a diagram to the common factor. For example, suppose we had in a circuit the combination shown in Figure 11. Both R_1 and C must have, at each instant, the same potential drop. Let us call that V_1. The inductor L and resistor R_2 both have, at each instant, the same current i. For the CR_1 section, refer the two current vectors $(i_{R_1})_0$ and $(i_C)_0$, to the common potential drop. The diagram is shown in Figure 12. The current in a resistor is always <u>in phase</u> with the potential drop across it, and the potential drop across a capacitor always <u>lags</u> behind the ac current in the capacitor by $\pi/2$ rad.

Refer the two potential drops in the LR_2 section to the common current i_0. The vector diagram is shown in Figure 13. The potential drop across R_2 is <u>in phase</u> with the current in the resistor. The current in an ideal inductor always <u>lags</u> behind the potential drop across it by $\pi/2$ rad. The only tricky part is to couple these two vector diagrams together. We do this by noting from the circuit diagram that

$$i(t) = i_{R_1}(t) + i_C(t),$$

or, if we add amplitudes or rms values, that i_0 is the <u>vector</u> sum of $(i_{R_1})_0$ and $(i_C)_0$. Going back to the first vector diagram, Figure 12, we can draw in i_0. We see from Figure 14 that i_0 leads $(i_{R_1})_0$ by

$$\phi = \tan^{-1}\left(\frac{(i_C)_0}{(i_{R_1})_0}\right) = \tan^{-1}\left(\frac{(V_1)_0\omega C}{(V_1)_0/R_1}\right) = \tan^{-1}(R_1\omega C).$$

Now, since we have i_0 on the first vector diagram, we can simply add the rest of the second vector diagram, rotating so that all vectors bear the proper relationship to i_0. See Figure 15.

Figure 14

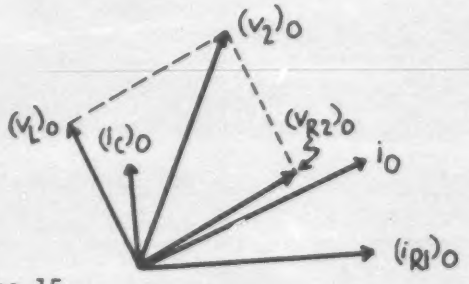


Figure 15

If the problem demands it, we can get $(V_2)_0$, the vector sum of $(V_L)_0$ and $(V_{R_2})_0$. We can even get the total amplitude of the potential drop,

$$\vec{V}_0 = (\vec{V}_1)_0 + (\vec{V}_2)_0.$$

Since the vector diagram gets messy by now, let us summarize by drawing two vector diagrams, one for currents and one for potential drops, Figures 16 and 17, respectively. By studying the diagram for potentials, you can see that

$$\theta = \tan^{-1}\frac{(V_2)_0}{(V_{R_2})_0} \quad \text{and} \quad \psi = \tan^{-1}\frac{(V_2)_0 \sin(\theta + \phi)}{(V_1)_0 + (V_2)_0 \cos(\theta + \phi)}.$$

Using these relationships, and equations such as

$$(V_L)_0 = (i_L)_0 \omega L, \quad (V_C)_0 = (i_C)_0/\omega C, \quad \text{and} \quad (V_R)_0 = (i_R)_0 R,$$

one can learn everything about the circuit, if the algebra does not get too messy. Remember, the instantaneous value of any of these quantities is the vertical component of the vector at time t, if the diagram is rotated counterclockwise with angular velocity ω.

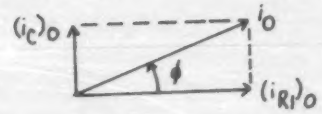


Figure 16

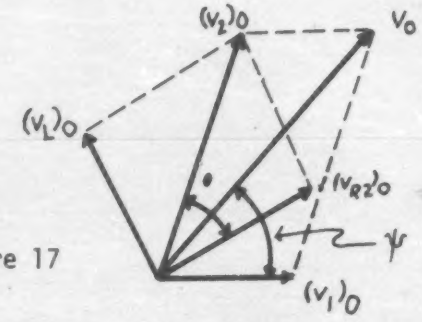


Figure 17

PROBLEM SET WITH SOLUTIONS

A(1). Figure 18 represents part of an ac circuit. The potential differences V_1 and V_2 are: $v_1(t) = v_{10} \sin(\omega t + \delta_1)$, $v_2(t) = v_{20} \sin(\omega t + \delta_1)$, where $v_{10} = 24.0$ V, $v_{20} = 16.0$ V, $\omega = 3.00 \times 10^3$ rad/s, $\delta_1 = -30°$, and $\delta_2 = 45°$.

(a) On a rotating vector diagram, show vectors for v_{10} and v_{20} at times $t = 0$; $t = (\pi/1.80) \times 10^{-4}$ s; and $t = (\pi/6) \times 10^{-3}$ s.

(b) Calculate the instantaneous values of $v_1(t)$, $v_2(t)$, and the total potential drop $v_1(t) + v_2(t)$ at the times specified in part (a).

(c) Determine the maximum value of v(t), and its phase with respect to either v_{10} or v_{20}.

(d) Plot v_1, v_2, and v as functions of time.

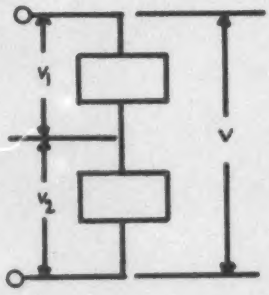


Figure 18

Solution

(a) At t = 0: see Figure 19(a). At $t = (\pi/1.80) \times 10^{-4}$,

$$\omega t = (3.00 \times 10^3)(\pi/1.80 \times 10^{-4}) = \pi/6 \text{ rad} = 30°$$

See Figure 19(b). At $t = \pi/6 \times 10^{-3}$,

$$\omega t = (3.00 \times 10^3)(\pi/6 \times 10^{-3}) = \pi/2 \text{ rad} = 90°.$$

See Figure 19(c).

(b) Component of v_0 along verical axis iv(t). See Figure 20. Instantaneous values add algebraically:

t = 0: $v_1(0) = v_{10} \sin \delta_1 = 24 \sin(-\pi/6) = -12.0$ V.

$v_2(0) = v_{20} \sin \delta_2 = 16 \sin(\pi/4) = 16(\sqrt{2}/2) = 11.3$ V.

$v(0) = v_1(0) + v_2(0) = -0.70$ V.

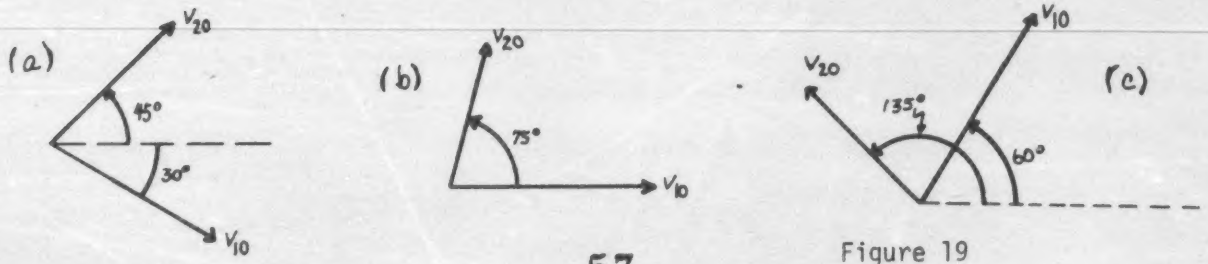


Figure 19

$t = \pi/1.80 \times 10^{-4}$: $v_1(\pi/1.80 \times 10^{-4}) = v_{10} \sin(\omega t + \delta_1) = v_{10} \sin(\pi/6 - \pi/6) = 0.$

This was obvious from the vector diagram. See Figure 21.

Figure 20

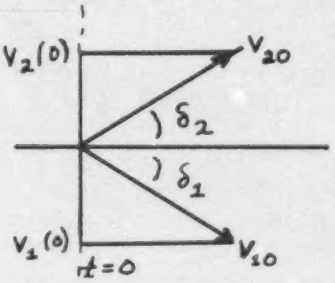
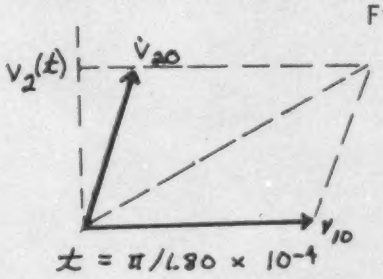


Figure 21



$v_2(\pi/1.80 \times 10^{-4}) = v_{20} \sin(\omega t + \delta_2) = v_{20} \sin(75°) = 12 \sin(75°) = 11.6$ V,

$v(\pi/1.80 \times 10^{-4}) = 11.6$ V.

$t = \pi/6 \times 10^{-3}$. $\omega t = \pi/2$ rad. See Figure 22.

$v_1(\pi/6 \times 10^{-3}) = v_{10} \sin(\omega t + \delta_1) = v_{10} \sin(\pi/3) = 20.8$ V.

$v_2(\pi/6 \times 10^{-3}) = v_{20} \sin(\omega t + \delta_2) = v_{20} \sin(3\pi/4) = 11.3$ V.

$v(\pi/6 \times 10^{-3}) = 32.1$ V.

Figure 22

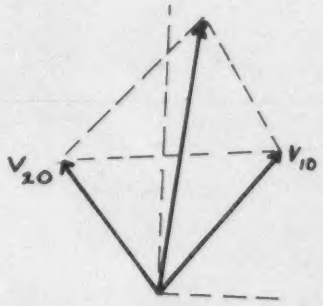
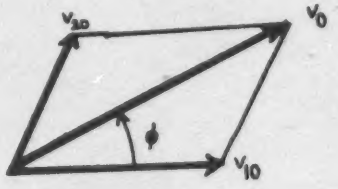


Figure 23



(c) Maximum values add like vectors. We can use any one of the vector diagrams. The easiest to use is Figure 21, since v_{10} has only one component:

$v_0^2 = v_{0x}^2 + v_{0y}^2$, $v_{0y} = v_{20} \sin 75° = 11.6$ V from part (b).

$v_{0x} = v_{10} + v_{20} \cos 75° = 28.1$ V, $v_0^2 = 924.17$, $v_0 = 30.4$ V.

$\phi = \tan^{-1}(v_{0y}/v_{0x}) = \tan^{-1}(11.6/28.1) = 22.4°$,

where ϕ is the phase angle by which v_0 <u>leads</u> v_{10}. See Figure 23.
(d) See Figure 24.

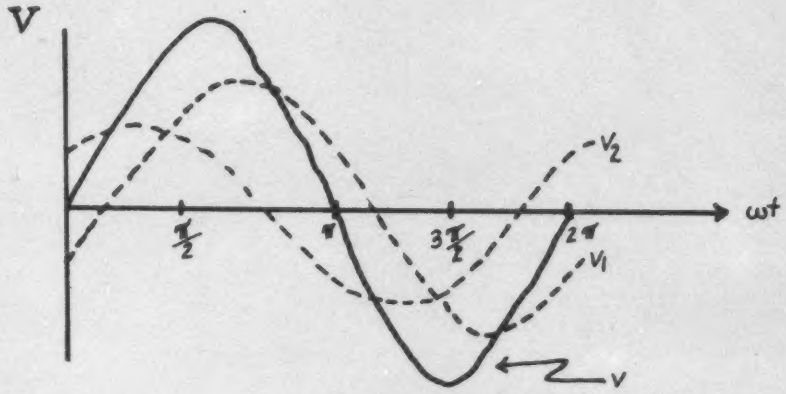


Figure 24

B(1-3). (a) In the circuit shown In Figure 25, draw a rotating vector diagram showing v_{RO}, v_{CO}, $\mathcal{E}_0$, and i_0.

(b) Calculate the current amplitude i_0, and v_{CO} and v_{RO}, in terms of $\mathcal{E}_0$, R, and C.

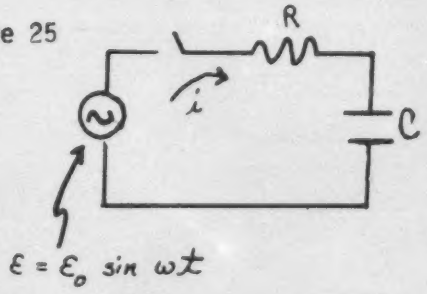
(c) Determine values of R and C such that the maximum potential difference across C is $(1/2)\mathcal{E}_0$ at a frequency of $f = \omega/2\pi = 10^6$ Hz.

(d) For the values chosen in part (c), determine the phase angle and draw a graph showing $v_C(t)$ and $\mathcal{E}(t)$ versus time.

(e) Calculate the maximum instantaneous power dissipated in the resistor.

(f) Compare the average power dissipated in the resistor with the average power supplied by the source of emf.

Figure 25



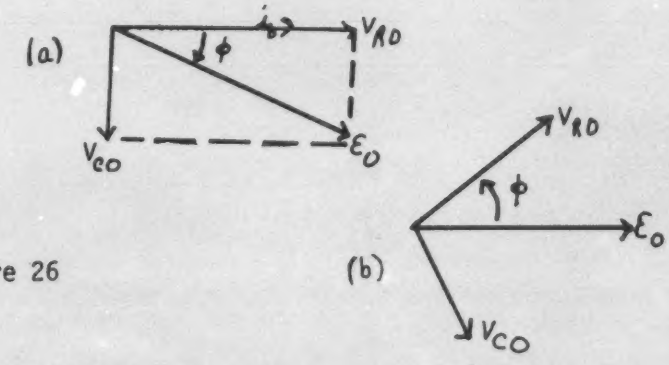


Figure 26

<u>Solution</u>

(a) Since i(t) is common to both R and C, refer the vectors to the current i. $v_R = iR$ is in phase with i, thus v_{RO} is parallel to i; v_C <u>lags</u> behind the current by $\pi/2$ rad (why?). See Figure 26. If you wish to rotate the diagram to time, t = 0, get $\mathcal{E}_0$ horizontal, as in Figure 26(b).

(b) $v_{RO} = i_0 R$, $v_{CO} = i_0/\omega C$,

$$\mathcal{E}_0^2 = i_0^2[R^2 + (1/\omega C)^2] \quad \text{or} \quad i_0 = \mathcal{E}_v/Z,$$

where

$$Z = [R^2 + (1/\omega C)^2]^{1/2}.$$

Does that make sense? Are dimensions right? As the plates of the capacitor come together, the capacitance increases. If they touch, the capacitor is shorted. See Figure 27.

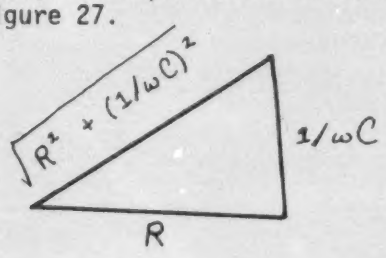


Figure 27

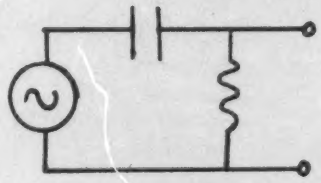


Figure 28

$\lim_{C \to \infty} i_0 = \mathcal{E}_0/R$. That looks OK.

$\lim_{R \to 0} i_0 = \mathcal{E}_0 \omega C = \mathcal{E}_0/X_C$, which also looks OK. Then $v_{CO} = i_0 X_C = \mathcal{E}_0 \sin\phi$, where $\phi = \tan^{-1}(v_{CO}/v_{RO}) = 1/R\omega C$ from the diagram. Then

$$v_{CO} = \mathcal{E}_0 \sin\phi = \mathcal{E}_0(1/\omega C)(1/\sqrt{R^2 + (1/\omega C)^2} = \mathcal{E}_0(1/\sqrt{1 + (R\omega C)^2}),$$

where v_{CO} is a function of frequency. The higher ω is, the smaller the potential drop across C. This circuit is called a "low-pass filter" if $v_C(t)$ is taken as the output. It "passes" low frequencies.

$$v_R = i_0 R = \mathcal{E}_0 \cos\phi = \mathcal{E}_0 \frac{R}{Z} = \mathcal{E}_C/[1 + C(\frac{1}{R\omega C})]^{1/2}.$$

v_R gets bigger as ω increases. If v_R is used as the output, it is called a "high-pass filter." As a high-pass filter, the circuit is usually written as shown in Figure 28.

(c) $v_{Co} = \mathcal{E}_0/\sqrt{1 + (R\omega C)^2} = \mathcal{E}_0/2$, therefore

$$1 + (R\omega C)^2 = 4, \qquad R = \sqrt{3}/\omega C = \sqrt{3}/(2\pi \times 10^6)C.$$

If we choose C = 1 nF = 10^{-9} F, then

$$R = \sqrt{3}/2\pi \times 10^3 = 276\ \Omega.$$

(d) See Figures 29 and 30.

$$\tan\phi = \frac{X_C}{R} = \frac{1}{R\omega C} = \frac{1}{(\sqrt{3}/2\pi \times 10^3)(2\pi \times 10^6)(10^{-9})} = \frac{1}{\sqrt{3}} = 0.577$$

$\phi = 0.523$ rad $\approx 30°$.

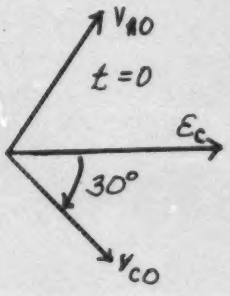


Figure 29

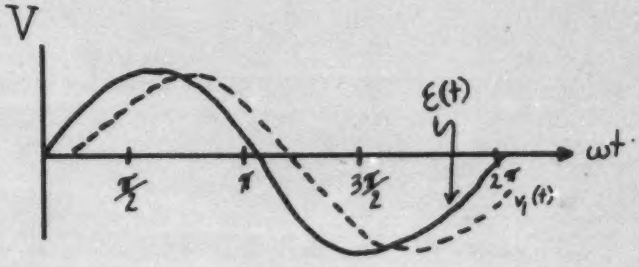


Figure 30

(e) $p_R(t) = [i(t)]^2R$. But $i(t) = i_0 \sin(\omega t + \theta)$, where $\theta = 90 - \phi = 60°$. The maximum power is

$$[p_R(t)]_{max} = [i_0^2 R \sin^2(\omega t + \theta)]_{max} = i_0^2 R = \frac{\mathcal{E}_0^2 R}{R^2 + (1/\omega C)^2}.$$

For $\omega = 2\pi \times 10^6$, $R = 276$, $C = 10^{-9}$, and

$$[p_R(t)]_{max} = \mathcal{E}_0^2(2.72 \times 10^{-3}).$$

C(1, 2, 3, 4). In the circuit shown in Figure 31,
$R = 3.00 \times 10^4\ \Omega$, $C = 10^{-8}$ F,
$L = 2.00$ H, $\omega = 10^4$ rad/s, and
$\mathcal{E}_0 = 25.0$ V.
(a) Calculate i(t).
(b) Calculate the average power supplied to the circuit.
(c) Plot $\mathcal{E}(t)$, i(t), and p(t) versus ωt.

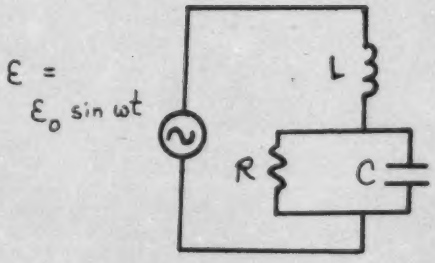


Figure 31

Solution

Begin by dividing the circuit into sections in which all elements have a potential drop or a current in common. See Figure 32. The RC combination has a common potential difference. Call it v_1.

$$v_1 = i_R R, \quad (1)$$

$$v_1 = i_C(i/\omega C), \quad (2)$$

Where i_R is in phase with v_1, and i_C leads v_1 by $\pi/2$ rad (why?). The total current i is the sum of i_R and i_C. The amplitude i_0 is the vector sum of i_{RO} and i_{CO}. (Of course, so far we do not know any of the currents.) The angle ϕ in Figure 33 is given by

$$\tan\phi = i_{CO}/i_{RO} = v_{i0}\omega CR/v_{i0} = \omega RC = 10^4(3.00 \times 10^4)(10^{-8}) = 3.00; \quad \phi = 71°34'. \quad (3)$$

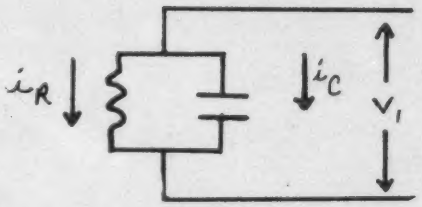


Figure 32

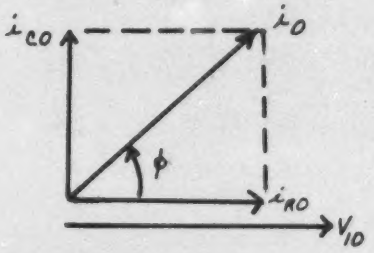


Figure 33

Now we have to hook up the rest of the circuit. The top and bottom elements have the current i_0 in common, therefore we draw a vector diagram, referring v_2 and v_1 to i_0. See Figures 34 and 35. v_2 leads i by $\pi/2$ rad (why?). The amplitude is

$$v_{20} = i_0 X_L = i_0 \omega L. \quad (4)$$

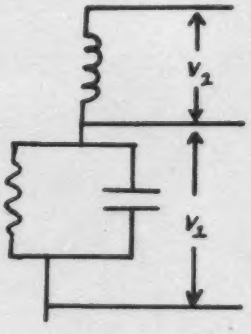


Figure 34

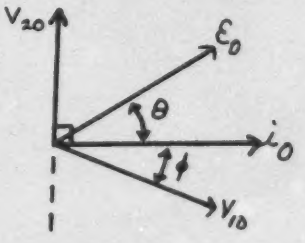


Figure 35

We already know from Eq. (3) that v_1 lags behind i_0 by a phase angle

$$\phi = \tan^{-1} \omega RC = 71°34'.$$

The vector sum of v_{10} and v_{20} is the amplitude of emf $\mathcal{E}_0$. We are given that $\mathcal{E}_0$ = 25.0 V. The angle θ is

$$\tan \theta = (v_{20} - v_{10} \sin \phi)/(v_{10} \cos \phi). \qquad (5)$$

Now let us take stock. We know $\mathcal{E}_0$, R, L, C, and ϕ. We do not know i_0, v_{10}, v_{20}, and θ. So let us start with $\mathcal{E}_0$, and work our way back. From the vector diagram,

$$\mathcal{E}_0^2 = \mathcal{E}_{0x}^2 + \mathcal{E}_{0y}^2 = (v_{10} \cos \phi)^2 + (v_{20} - v_{10} \sin \phi)^2. \qquad (6)$$

If we can get v_{10} and v_{20} in terms of i_0, we can solve for i_0, and also get θ:

$$v_{20} = i_0 \omega L, \quad v_{10} = i_R R = i_0 (\cos \phi) R.$$

The last step comes from the first vector diagram, Figure 33. Equation (6) now is

$$\mathcal{E}_0^2 = i_0^2 (\omega L)^2 \cos^2 \phi + i_0^2 [R \cos \phi - \omega L \sin \phi]^2. \qquad (6')$$

$$i_0^2 = \mathcal{E}_0^2 / \{(\omega L)^2 + [R \cos \phi - \omega L \sin \phi]^2\}$$

$$= (25)^2 / \{(2 \times 10^4)^2 + [(3.00 \times 10^4)(\cos 71°34') - (2 \times 10^4)(\sin 71°34')]^2\}$$

$$= 1.13 \times 10^{-3} \text{ A}. \qquad (7)$$

We know, from Eq. (4), that $v_{20} = i_0 \omega L$ = 22.6 V and $v_{10} = i_0 R \cos \phi$ = 10.7 V. Equation (5) then gives us θ:

$$\tan \theta = 3.68; \quad \theta = 74.8°.$$

Finally,

$$i = i_0 \sin(\omega t + \theta) = (1.13 \times 10^{-3}) \sin(\omega t + 74.8°)$$

$$= (1.13 \times 10^{-3}) \sin(\omega t + 1.31) \quad \text{if the angle is in radians.}$$

(b) $p_{av} = \mathcal{E}_{rms} i_{rms} \cos \phi = (1/2) \mathcal{E}_0 i_0 \cos \phi = 4.5 \times 10^{-3}$ W.

(c) See Figure 36.

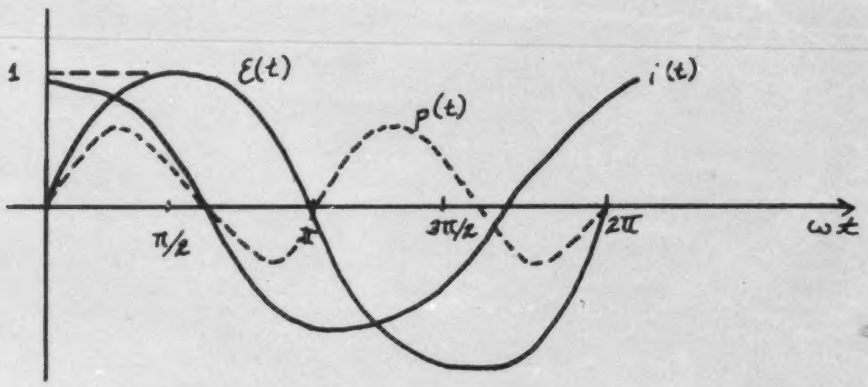


Figure 36

Problems

D(1). In Figure 37 are sketched two potential differences as functions of time.

(a) State whether v_2 leads or lags v_1.

(b) Draw a vector diagram for v_1 and v_2 at an instant of time that you choose. State the instant you choose.

(c) In terms of ω (assumed known) and the other given constants, write algebraic expressions for $v_1(t)$ and $v_2(t)$.

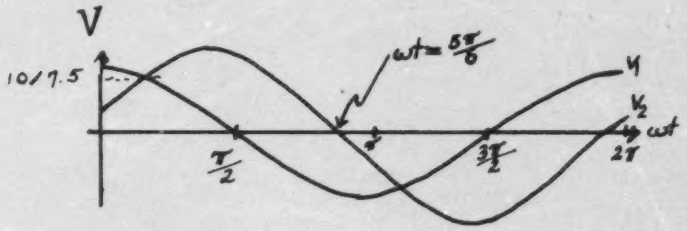


Figure 37

E(1-4). (a) In the circuit shown in Figure 38, draw a rotating vector diagram showing v_{RO}, v_{LO}, $\mathcal{E}_0$, and i_0.

(b) Calculate the current amplitude i_0, and v_{RO} and v_{LO}, in terms of $\mathcal{E}_0$, R, L, and ω.

(c) Determine values of R and L such that the maximum potential difference across L is $(1/2)\mathcal{E}_0$ at a frequency of $f = \omega/2\pi = 2.00 \times 10^4$ Hz.

(d) For the values chosen in part (c), determine the phase angle and draw a graph showing $v_1(t)$ and $\mathcal{E}(t)$ versus time.

(e) Calculate the maximum instantaneous power dissipated in the resistor.

(f) Compare the average power dissipated in the resistor with the average power supplied by the source of emf.

F(1-4). In the circuit shown in Figure 39, i_0 = 10.0 mA, f = 10^3 Hz, L_1 = 10.0 mH, L_2 = 250 mH, C = 0.100 μF, and R = 1.00 kΩ.

(a) Draw phase diagrams for the L_1-C segment and the L_2-R segment of the circuit.

(b) Calculate $v_{L_1 0}$, v_{C0}, $v_{L_2 0}$, v_{R0}.

(c) Calculate $\mathcal{E}_0$.

(d) At $\omega t = 2\pi$, calculate the energy stored in L_1, C, and L_2.

(e) At $\omega t = 3\pi/2$, calculate the energy stored in L_1, C, and L_2.

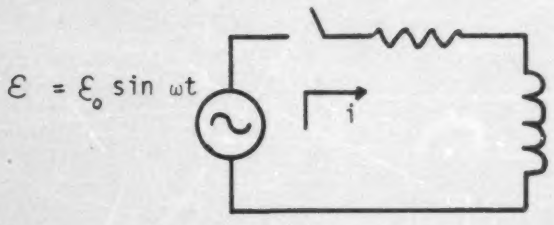


Figure 38

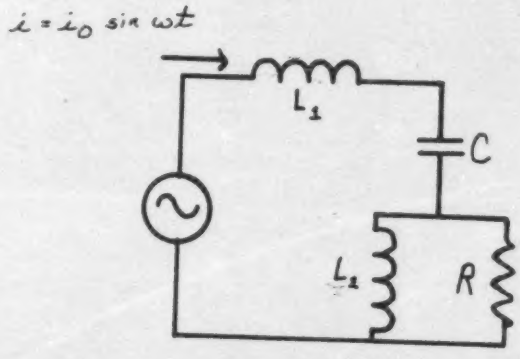


Figure 39

Solutions

D(1). (a) v_2 lags v_1 by $\pi/3$.

(b) See Figure 40, at t = 0.

(c) $v_1(t) = 10.0 \sin(\omega t + \pi/2) = 10.0 \cos \omega t$.

$v_2(t) = 7.5 \sin(\omega t + \pi/2 - \pi/3) = 7.5 \cos(\omega t - \pi/3)$.

E(1-4). (a). See Figure 41.

(b) $i_0 = \mathcal{E}_0/\sqrt{R^2 + (\omega L)^2}$. $v_{R0} = \mathcal{E}_0 R/\sqrt{R^2 + (\omega L)^2}$. $v_{L0} = \mathcal{E}_0 \omega L/\sqrt{R^2 + (\omega L)^2}$.

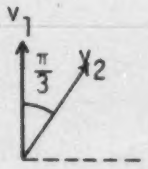


Figure 40

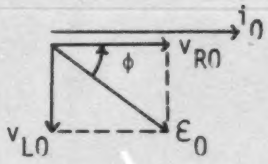


Figure 41

(c) $R = 3.46 \times 10^4 L$; e.g., if $L = 0.200$ H, $R = 6.92 \times 10^3\ \Omega$.

(d) $\phi = 30°$. See Figure 42.

(e) $(P_r)_{max} = i_0^2 R = \mathcal{E}_0^2 R/[R^2 + (\omega L)^2]$.

(f) $(P_R)_{av} = (1/2) i_0^2 R = \mathcal{E}_0^2 R/2[R^2 + (\omega L)^2]$.

$(P_\mathcal{E})_{av} = 8_0^2 R/2[R^2 + (\omega L)^2]$.

F(1-4). (a) See Figure 43. $\phi = 57.5° \approx 1.00$ rad.

(b) $v_{L_1 0} = 0.628$ V; $v_{C0} = 15.9$ V; $v_{L_2 0} = 13.2$ V; $v_{R0} = 13.2$ V.

(c) $\mathcal{E}_0 = 11.9$ V.

(d) $E(L_1) = 0$; $E(C) = 1.26 \times 10^{-5}$ J; $E(L_2) = 2.57 \times 10^{-6}$ J.

(e) $E(L_1) = 0.50 \times 10^{-6}$ J; $E(C) = 0$; $E(L_2) = 6.32 \times 10^{-6}$ J.

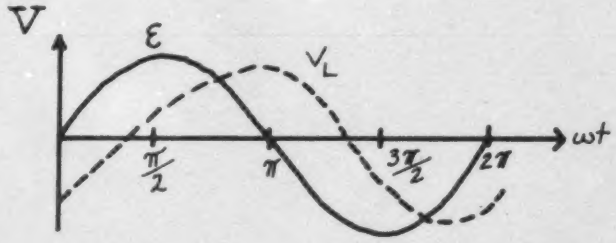


Figure 42

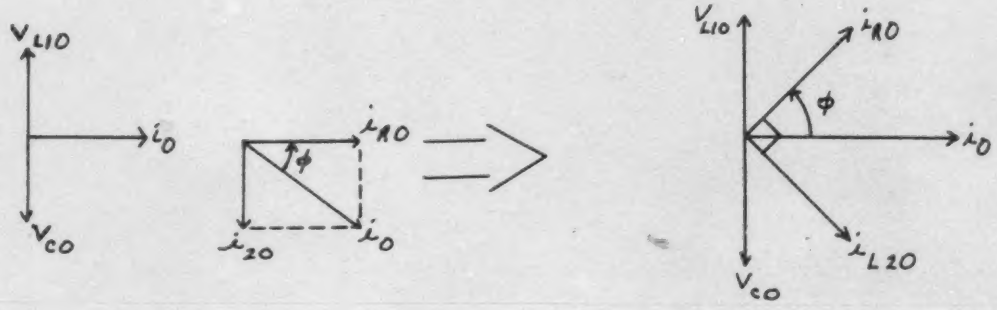


Figure 43

PRACTICE TEST

1. For the circuit shown in Figure 44, draw a vector diagram illustrating the relative phase relations among the currents, i_{R_0}, i_{L_0}, i_{C_0}, and i_0.

2. Calculate i_R, i_L, i_C, and i as functions of time. Let

   $\mathcal{E}_0 = 30.0$ V, $R = 2.00$ kΩ, $L = 5.0$ mH, $C = 2.00$ μF, $\omega = 2.00 \times 10^4$ rad/s.

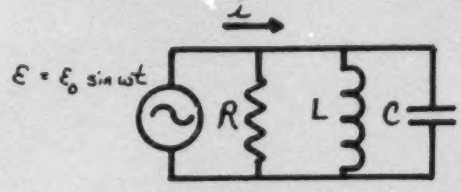
3. Calculate the average power input to the resister, the inductor, and the capacitor. Calculate the average power supplied by the source of emf.

4. Describe qualitatively the energy flow in the circuit.

5. How would your answers change if the angular frequency were changed to

   $\omega_0 = 1/\sqrt{LC} = 1.00 \times 10^4$ rad/s?

Figure 44



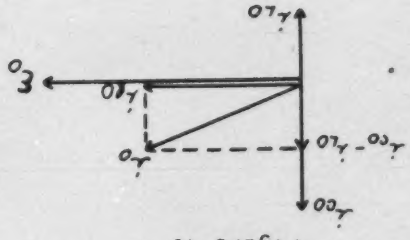


Figure 45

Practice Test Answers

1. See Figure 45.

2. $i_R(t) = 15.0 \times 10^{-3} \sin(2.00 \times 10^4 t)$; $i_L(t) = 0.300 \sin(2.00 \times 10^4 t - \pi/2)$.

   $i_C(t) = 1.20 \sin(2.00 \times 10^4 t + \pi/2)$; $i(t) = 0.90 \sin(2.00 \times 10^4 t + 89°)$.

3. $(P_R)_{av} = 0.225$ W; $(P_L)_{av} = 0$; $(P_C)_{av} = 0$; $(P_\mathcal{E})_{av} = 0.225$ W.

4. Source supplies energy that is dissipated in the resistor, as heat. Some of the remaining energy oscillates back and forth between the electric field of the capacitor and the magnetic field of the inductor. The rest is alternately pumped into the source and is supplied by the source.

5. At the resonance frequency, $i_0 = i_R = \mathcal{E}/R$. The phase angle is zero. The total energy in the LC part of the circuit remains constant, simply oscillating back and forth between the capacitor and the inductor.

ALTERNATING-CURRENT CIRCUITS

Date _____

Mastery Test Form A

pass recycle

1 2 3 4

Name _____

Tutor _____

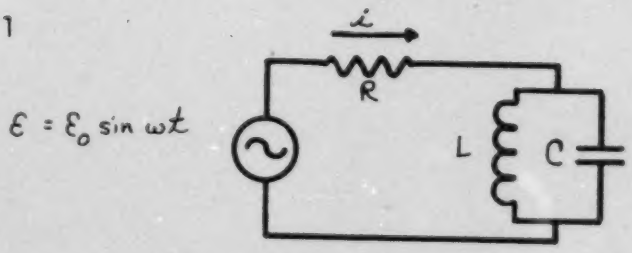
1. In the circuit shown in Figure 1, calculate the currents as a function of time. Include in your solution a phase diagram showing all relevant phases. Assume that $\mathcal{E}_0$, ω, R, L, and C are known. Plot i(t) and $\mathcal{E}(t)$ as functions of ωt.

2. Given that ω is 10^6 rad/s, $\mathcal{E}_0$ = 15.0 V, R = 10.0 kΩ, determine values of L and C that will satisfy the resonance condition. What happens to the amplitude of i under that condition?

3. With the values of L and C determined in Problem 2, calculate the maximum energy in the magnetic field of the inductor, and the maximum energy in the electric field of the capacitor. Describe how these energies vary with time.

4. Calculate the average power delivered to the circuit under the conditions of Problem 2.

Figure 1



ALTERNATING-CURRENT CIRCUITS

Date _____

Mastery Test Form B

pass recycle

1 2 3 4

Name _____

Tutor _____

The following problems apply to the circuit shown in Figure 1. Assume $\mathcal{E}_0$, ω, R, L and C are known.

1. Divide the circuit into sections that have a common current or potential. Draw a vector diagram for each section.
2. What is the phase angle between the emf $\mathcal{E}$ and the potential drop across the resistor V_R? Does V_R lead or lag $\mathcal{E}$? What is the phase angle between the current i and the emf $\mathcal{E}$? You may express your answer in terms of the (unknown) currents i_1 and i_2 if you wish.
3. Calculate the average power dissipated in the resistor, in terms of $\mathcal{E}_0$, R, L, C, and ω.
4. Describe the energy flow in the circuit, qualitatively, and as quantitatively as you can.

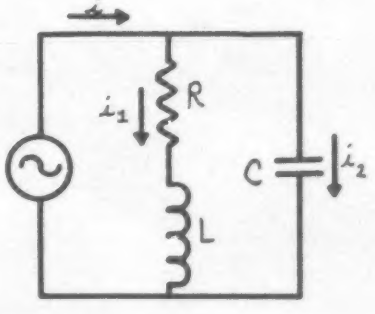


Figure 1

$\mathcal{E} = \mathcal{E}_0 \sin \omega t$

ALTERNATING-CURRENT CIRCUITS

Date _____

Mastery Test Form C

pass recycle

1 2 3 4

Name _____ Tutor _____

1. In the circuit shown in Figure 1, draw a vector diagram showing all currents and potential differences. Calculate the current i as a function of time. Let $\mathcal{E}_0$ = 1.00 V, ω = 377 rad/s (or f = 60 Hz), R = 500 Ω, L = 4.0 H, and C = 2.00 μF.

2. Calculate the power delivered by the source as a function of time. Calculate the power into the field of the capacitor as a function of time.

3. Calculate the average power delivered to the circuit. What would you do to L, or C, or the product LC, to maximize the power delivered?

4. Describe the energy flow as a function of time.

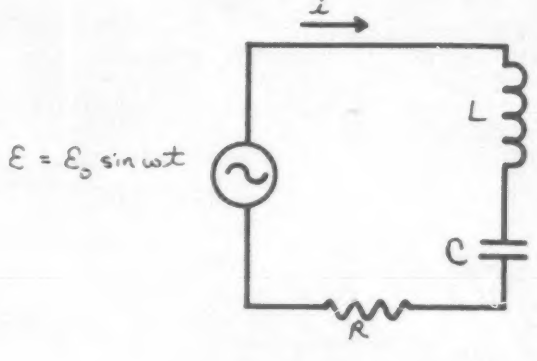


Figure 1

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Diagram may be done in parts, but all phases should be properly indicated. Figure 50 is plotted for positive ϕ. Be sure student knows the difference between positive and negative phase.

Solution: See Figure 49. $v_{1O} = i_{LO}\omega L = i_{CO}/\omega C$. (A1)

$$v_{RO} = i_0 R. \quad (A2)$$

$$i_0 = i_{CO} - i_{LO}. \quad (A3)$$

From Eq. (A1), $i_{CO} = i_{LO}\omega^2 LC$. From Eq. (A3),

$$i_0 = i_{LO}(\omega^2 LC - 1); \quad (A4)$$

$$\phi = \tan^{-1}(v_{1O}/v_{RO}) = \tan^{-1}(i_{LO}\omega L/i_0 R) = \tan^{-1}[\omega L/R(\omega^2 LC - 1)]. \quad (A5)$$

From Eqs. (A4) and (A1),

$$i_0 = i_{LO}(\omega^2 L^{\wedge} - 1) = (v_{1O}/\omega L)(\omega^2 LC - 1) = (\mathcal{E}_0 \sin \phi/ L)(\omega^2 LC - 1); \quad (A6)$$

$i = i_0 \sin(\omega t + \phi)$. See Figure 50.

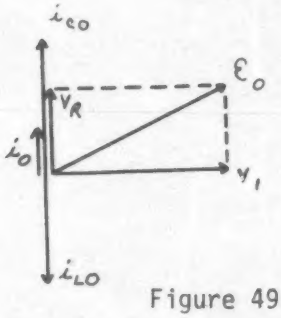


Figure 49

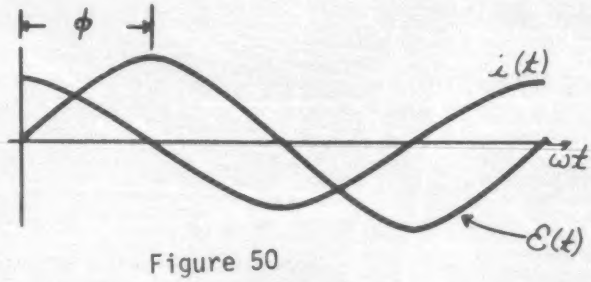


Figure 50

2. What To Look For: Any combination of L and C such that $LC = 10^{-12}$ is OK.

Solution: Resonance occurs when $\omega = 1/\sqrt{LC}$, therefore

$$LC = 1/\omega^2 = 10^{-12} \text{ s/rad}.$$

Let L = 1.00 mH, C = 1 nF, for example?

$$i_0 = \mathcal{E}_0 \sin \phi(\omega C - 1/\omega L) = \mathcal{E}_0(\omega C - 1/\omega L)\{\omega L/[R^2(\omega^2 LC - 1)^2 + (\omega L)^2]^{1/2}\}.$$

If $\omega = 1/\sqrt{LC}$, $i_0 = \mathcal{E}_0 \times 0 = 0$.

3. What To Look For: There is no current from the source at resonance. Know energy of an inductor and a capacitor. The maximum energies in L and C are the same at resonance.

Solution: $(E_L)_{max} = (1/2)Li_{L0}^2$. Since i_0 is zero, the potential difference across the inductor and capacitor is the potential difference across the source:

$$\mathcal{E}_0 = v_{10} = i_{L0}\omega L, \qquad i_{L0} = \mathcal{E}_0/\omega L;$$

$$(E_v)_{max} = (1/2)Li_{L0}^2 = \mathcal{E}_0^2/2\omega^2 L, \qquad \omega^2 = 1/LC.$$

Therefore,

$$(E_L)_{max} = (1/2)\mathcal{E}_0^2 C = (1/2)(15)^2(10^{-9}) = 1.125 \times 10^{-7}\ \text{J},$$

$$(E_C)_{max} = (1/2)Cv_{10}^2 = (1/2)C\mathcal{E}_0^2 = 1.125 \times 10^{-7}\ \text{J}.$$

Since $\phi = \pi/2$, E_L is a maximum when E_C is zero, and vice versa. Energy leaves the collapsing magnetic field, and goes into building the electric field, etc.

4. Solution: Since the current i_0 is zero, the power delivered to circuit:

$$P_{av} = (1/2)\mathcal{E}_0 i_0 \cos\phi = 0.$$

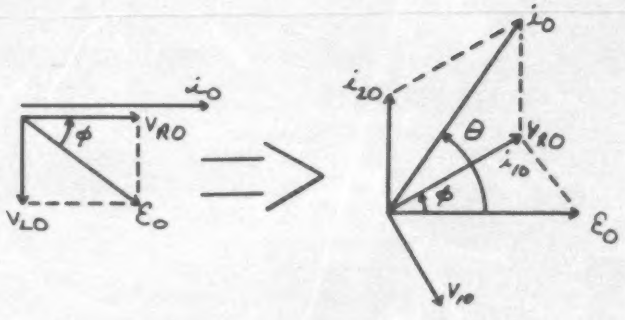
Current oscillates in the LC branch, but no power is dissipated, and none is delivered by the source.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: Correct phase relations. Ask student about omitted quantities.

Solution: See Figure 51.

Figure 51



2. What To Look For: Any of the relations below are OK. Algebra not as important as beginning with correct relationships.

Solution: $\phi = \tan^{-1}(v_{L0}/v_{R0}) = \tan^{-1}[i_{10}\omega L/i_{10}R] = \tan^{-1}(\omega L/R)$. v_R leads $\mathcal{E}$. The angle between i and $\mathcal{E}$ is θ.

$$\theta = \tan^{-1}\left[\frac{(i_{20})_y + (i_{10})_y}{(i_{10})_x}\right] = \tan^{-1}\left[\frac{i_{20} + i_{10}\sin\phi}{i_{10}\cos\phi}\right]$$

$$= \tan^{-1}\left[\frac{\mathcal{E}_0\omega C + (\mathcal{E}_0/R)\cos\phi\sin\phi}{(\mathcal{E}_0/R)\cos^2\phi}\right] = \tan^{-1}\left[\frac{\omega(CR^2 + \omega^2L^2) + \omega L}{R}\right].$$

3. What To Look For: Knowledge that rms voltages and currents are $1/\sqrt{2}$ times amplitudes. Look for correct place to start in P_R calculation.

Solution: $i_{10} = v_{R0}/R = (\mathcal{E}_0\cos\phi)/R = \mathcal{E}_0/(R^2 + \omega^2L^2)^{1/2}$.

$$(P_R)_{av} = (v_R)_{rms}(i_1)_{rms}\cos\phi = (1/2)v_{R0}i_{10}\cos\phi = \mathcal{E}_0^2R^2/2(R^2 + \omega^2L^2)^{3/2}.$$

4. Solution: Energy will oscillate back and forth between magnetic and electric fields. Energy dissipated by the resistor will be supplied by the source. In general, some energy will alternately enter and leave the source from L or C. Quantitatively,

$$P_L(t) = i_1 v_L = i_1^2 \omega L \sin(\omega t + \phi) \sin(\omega t + \phi - \tfrac{7}{2})$$

$$= -\frac{\mathcal{E}_0^2 R^2 \omega L}{(R^2 + \omega^2 L^2)} \sin(\omega t + \phi) \cos(\omega t + \phi) = -\frac{\mathcal{E}_0^2 R^2 \omega L}{2(R^2 + \omega^2 L^2)} \sin[2(\omega t + \phi)].$$

$$P_C(t) = v_C i_2 = \mathcal{E}_0^2 \omega C \sin \omega t \sin(\omega t + \pi/2) = (1/2)\mathcal{E}_0^2 \omega C \sin[2(\omega t)].$$

$$P_R(t) = v_R i_1 = i_{10}^2 R \sin^2(\omega t + \phi).$$

To plot these would be very messy. It is important that P_L and P_C are out of phase, and there is an oscillation in energy.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: Correct signs of all phases. All angles correct. Arithmetic not so important. Proper setup is important.

   Solution: See Figure 52.

$$i = \frac{\mathcal{E}}{z} = \frac{\mathcal{E}_0 \sin(\omega t + \phi)}{[R^2 + (\omega L - 1/\omega c)^2]^{1/2}}; \qquad \phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right) = 20.0°.$$

$$i = \frac{156 \sin(\omega t + 20°)}{(500)^2 + (377(4) - 1/[377(2 \times 10^{-6}])^2\ ^{1/2}} = 0.293 \sin(\omega t + 20°).$$

2. Solution: $P_{\mathcal{E}}(t) = \mathcal{E}(t)i(t) = \mathcal{E}_0 i_0 \sin \omega t \sin(\omega t + \phi) = 45.7 \sin \omega t \sin(\omega t + \phi)$.

$$P_C(t) = v_C(t)i(t) = (i^2/\omega C) \sin(\omega t - \pi/2) \sin(\omega t + \phi) = -(i^2/\omega C) \cos \omega t \sin(\omega t + \phi)$$

$$= -114 \cos \omega t \sin(\omega t + \phi).$$

$$(P_{\mathcal{E}})_{av} = (1/2)\mathcal{E}_0 i_0 \cos \phi = (1/2)(156)(0.293) \cos 20° = 21.4 \text{ W}.$$

3. Maximize i_C and $\cos \phi$ by setting $\sqrt{LC} = 1/\omega$. Maximum energies in the inductor and capacitor are not the same, except in resonance. In this case,

$$(E_L)_{max} = (1/2)Li_0^2 = 0.172 \text{ J}. \qquad (E_C)_{max} = (1/2)Cv_{C0}^2 = (1/2)C(i_0^2/\ ^2C^2) = 0.151 \text{ J}.$$

The 0.151 J is transferred from the electric field of the capacitor to the magnetic field of the inductor. Energy is dissipated in the resistor, with the maximum value of power dissipation being $(P_R)_{max} = (1/2)i_0^2R = 21.5$ W, with an average value of 10.7 W. Some energy (0.021 J max) is alternatively put back into the source, and delivered by it to the electric field of the capacitor.

Figure 52